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# QUANTUM GRAVITY AND QUANTUM ELECTRODYNAMICS

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**Abstract**

The Author is of the opinion that a sound piece of theoretical physics arises when one tries to understand a certain striking regularity in Nature. A classical example is the General Theory of Relativity which is the result of meditation on the equality of inertial and gravitational mass. In "Quantum Gravity", whatever this term is supposed to mean, there is nothing comparable to the equality of inertial and gravitational mass. By contrast, in Quantum Electrodynamics (not Q.E.D.!) there is a phenomenon every bit as striking as this equality: it is the universality of charge quantization. Electron's and proton's electric charges are equal with experimental accuracy 1:$10^{-20}$ while inertial and gravitational masses are equal with experimental accuracy 1:$10^{-12}$. The electric charge is a manifestation of substantiality of matter. By substantiality we mean something which is or is not there, regardless

of philosophical uncertainties which surround quantum mechanics. Therefore the road to "quantum gravity" should go through the understanding of charge quantization. This is the ground upon which the theoretical prowess, if it exists at all, should be tested first. As Descartes puts it, most people prefer to speculate upon difficult things rather than to arrive at the truth in simple things. This fatal error has to be abandoned, if we are to make a real progress in theoretical physics.

## 1 Introduction

The term "quantum gravity" is frequently used nowadays. There is a journal "Classical and Quantum Gravity", there are many books with "quantum gravity" in their titles. It is tempting to assume that when so many people write about "quantum gravity", they must know what they are writing about. Nevertheless, everyone will agree that there is not a single physical phenomenon whose explanation would call for "quantum gravity". One could say, of course, that everything around us is a manifestation of "quantum gravity" because everything has some mass and mass must ultimately have a quantum mechanical origin. This may very well be true but statements of such generality do not belong to theoretical physics. Theoretical physics consists in building mathematical models of certain selected aspects of physical reality. In choosing a model we have to have some guiding principle. The world of mathematical structures is just too vast to be inspected at random. We need some guiding principle to organize our mathematical knowledge and our physical intuition. A paradigmatic example is provided by General Theory of Relativity. By universal agreement General Theory of Relativity is a masterpiece and an example of the immense power of speculative thinking. But it is well known that this beautiful theory arose from meditations upon a physical phenomenon well known to Newton: exact equality of inertial and gravitational mass.

If we take the creation of General Theory of Relativity as a model to be followed, then we need a phenomenon which would play the role of equality of inertial and gravitational mass. The Author is of the opinion that such a phenomenon does exist: it is the exact (mathematical) equality of electron's and proton's electric charges. This phenomenon is so familiar that many people mistakenly take it as something obvious or already explained. The very term "elementary charge", commonly used in all textbooks, suggests an understanding which is not there: there is only the charge of electron and the charge of proton. Charges of all electrons must be indeed the same, otherwise one could use the value of the electric charge as a label which distinguishes a given electron from another one. But this argument does not apply to electrons and protons; their charges could very well be different. If neutrino can carry out an inordinately small mass, why it should not be able to carry out an inordinately small charge?

Nothing illustrates better the widespread misconception than suggestions recently made by a group of astrophysicists (I will not quote them because I think they are mistaken) that the fine structure constant has changed during the last ten billion years. The suggested rate of change is roughly 1/100 000 during the first few billion years. Later on, we are told, the constant mysteriously ceased to change. At the same time the very term "fine structure constant" assumes the exact equality of electron's and proton's electric charges. This means that two independent functions of cosmic time — the electron fine structure constant and the proton fine structure constant are said to change at a rate of 1/100 000 per billion years while sticking together with the phantastic precision $1{:}10^{-20}$. The very absurdity of this picture forces us to conclude that the suggested change is not there, as more recent observations do confirm.

One has the impression that the problem of universality of charge quantization is so fundamental that it must have a conceptually simple solution, like General Theory of Relativity which is a conceptually simple solution of the problem of exact equality of inertial and gravitational masses. Conceptual simplicity may very well coexist with technical complexity, this is precisely the case in General Theory of Relativity. "Mathematical tricks" (Pauli's expression) should not be allowed to play a role in the explanation of a problem of such proportions. The explanation we are looking for should be a "principle — theory" (Einstein's expression from the essay "What is the Theory of Relativity").

The present Author formulated in 1987 Quantum Mechanics of the Electric Charge [1], a theory which combines conceptual simplicity with a very modest amount of technical complexity. The theory does allow to explain the existence of only one elementary charge and non-existence of magnetic monopoles. Moreover, the structure of the theory does depend on the numerical value of the fine structure constant $\hbar c/e^2$. Below I shall state all the physical principles underlying the Quantum Mechanics of the Electric Charge. My goal is to exhibit the clarity and simplicity of physical principles.

## 2 The inequality of Berestetskii, Lifshitz, and Pitaevskii

Berestetskii, Lifshitz, and Pitaevskii [2] say that the electromagnetic field $F_{\mu\nu}$ is approximately classical if ($\hbar = 1 = c$)

$$\sqrt{F_{01}^2 + F_{02}^2 + F_{03}^2}\,(\Delta t)^2 \gg 1, \tag{1}$$

where $\Delta t$ is the time interval over which the field can be averaged without being significantly changed. For a static Coulomb field this time interval is infinite and therefore, conclude Berestetskii, Lifshitz and Pitaevskii, a static field is always classical. This conclusion, in view of the phenomenon of charge quantization, is completely absurd, but Berestetskii, Lifshitz, and Pitaevskii (B.L.P.) leave it without comment.

To the best of my knowledge, the book by B.L.P. is the only modern source which states the inequality (1). However, this inequality has a very long history, now apparently forgotten. The history goes back to the famous paper by Landau and Peierls and is told by Pauli [3]. It is interesting to see from Pauli's comments that the founders of Quantum Mechanics already saw here a place for the numerical value of the fine structure constant. It is also interesting to see that Pauli, with some hesitation, endorses the conclusion of B.L.P.

Some time ago I proposed a solution of the obvious contradiction between the conclusion of B.L.P. and the phenomenon of charge quantization. One can see from Pauli's comments that this solution is completely original and was never contemplated by the founders of Quantum Mechanics. One has to note that the electric charge, on the strength of the Gauss law, "lives" at the spatial infinity, $r = \left[(x_1)^2 + (x_2)^2 + (x_3)^2\right]^{1/2} = \infty$. At the spatial infinity the entire eternity of time, formally infinite, is limited by the opening of the light cone, $-r < t < r$; thus $\Delta t = 2r$ and the B.L.P. inequality for the Coulomb field with the total charge $Q$ takes on the form

$$\frac{|Q|}{r^2}(2r)^2 \gg 1 \tag{2}$$

i.e.

$$|Q| \gg \frac{1}{4} = \frac{\sqrt{137}}{4\sqrt{137}} = 2.93e, \tag{3}$$

which obviously means that charges of the order of the elementary charge are not classical objects. Charge quantization which, as all concerned seem to agree, cannot be obtained in Q.E.D., is an emergent phenomenon, separated from Q.E.D. by the nontrivial limit $r = \infty$ taken first.

## 3 The electromagnetic field at the spatial infinity

Having identified the spatial infinity as the proper arena for the quantum theory of the electric charge, we have to investigate the electromagnetic field there. This was done by Alexander and Bergmann [4]. Below I summarize Bergmann's analysis as completed in my contribution to the Yakir Aharonov Festschrift. I use four-dimensional tensor notation which is more transparent than Bergmann's three-dimensional notation.

At the spatial infinity the electromagnetic field is completely described by a single, homogeneous of degree zero and gauge invariant solution of the d'Alembert equation. If the potential $A_\mu(x)$ is assumed to be homogeneous of degree $-1$, which is natural, then this single, homogeneous of degree zero and gauge invariant function is equal to

$$x^\mu A_\mu(x), \quad \Box\left(x^\mu A_\mu(x)\right) = 0. \tag{4}$$

## 4 The theory

At the spatial infinity the total electric charge is a quantum object; this follows from the B.L.P. inequality. Therefore there must be something which does not commute with the total electric charge. But the only thing left at the spatial infinity is the function $x^\mu A_\mu(x)$. All information about charged matter is erased by the masses of charged particles. Therefore, the only choice left is

$$[Q,\ x^\mu A_\mu(x)] \neq 0. \tag{5}$$

This is the desired universality of charge quantization: at the spatial infinity there is only one candidate for the coordinate canonically conjugate with the total electric charge.

## 5 The status of the fine structure constant

I am perfectly well aware that no one will accept the above theory unless I am able to show that it does allow to fix the numerical value of the fine structure constant $1/e^2$ One can show that the structure of the theory does depend on the numerical value of the constant $e^2$. In particular the values $e^2 = \pi$ and $e^2 = \pi/2$ are critical and separate kinematically different sectors of the theory.

However, the very possibility of contemplating this theory shows the ambivalence of the notion of emergence, as discussed by Weinberg and Anderson in their famous debate. The theory described above, whether true or false, is emergent with respect to Q.E.D. because it is separated from Q.E.D. by the nontrivial limit $r = \infty$ taken first. At the same time this theory is a language which allows to speak about the fine structure constant. If this theory is true then the fine structure constant $1/e^2$ is, like $\pi$, a property of space-time. We mean, of course, the abstract pseudoeuclidean space-time of special relativity. As such the fine structure constant precedes the real (cosmological) space-time and is the most non-emergent quality of the world. In the words of Kepler (I quote after Pauli [5]) "the mathematical principles according to which the corporeal world was to be created are coeternal with God".

## References

[1] A. Staruszkiewicz, "Quantum Mechanics of Phase and Charge and Quantization of the Coulomb Field", preprint TPJU — 12/87, June 1987. The preprint is still available electronically at KEK. A spoiled version of this preprint was published in Annals of Physics (N.Y.) **190**, 354 (1989). The imperfection present in this publication is $\exp(iS(x))$, where $S(x)$ is an operator valued distribution. The presence of this imperfection can be only explained as a result of corruption brought about by contact with people who do such things habitually and without bad conscience.

[2] V.B. Berestetskii, E.M. Lifshitz and L.P. Pitaevskii, Relativistic Quantum Theory, Vol. 4 of Course of Theoretical Physics, Pergamon Press, Oxford 1971, Eq. (5.2) on page 14.

[3] W. Pauli, General Principles of Quantum Mechanics, Springer Verlag 1980, page 187.

[4] M. Alexander and P.G. Bergmann, Foundations of Physics **14**, 925 (1984).

[5] W. Pauli, Writings on Physics and Philosophy, Springer Verlag 1994, page 228.