Comment on
CONSIDERATIONS CONNECTED WITH THE
WORDS “IS” AND “EXISTS”
by Jerzy A. Janik
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Abstract
The relation between quantum collapse and consciousness in the context of quantum cosmology of closed universe is analyzed.
1 Motivation

In his work professor J.A. Janik is trying to answer the question of what does it mean for a physicist that a given object “is” and when it can be said that a given object not only “is” but also “exists” in time. In doing so J.A. Janik is touching the existential aspect of quantum physics. One of the main questions J.A. Janik ponders about is whether the collapse superposition of states at the moment of measurement needs the involvement of a conscious observer or a certain decoherent procedure is enough. I will try to answer these questions within the quantum cosmology of a closed universe.

2 Universal cosmic time

In 1929 an American astronomer Edwin Hubble discovered that distant galaxies move away from us at a speed directly proportional to the distance from the Earth. The speed depends only on the distance from the Earth and not on the direction in which a given galaxy is observed. If $\vec{r}(t)$ denotes the position vector at time $t > 0$ of any arbitrarily chosen distant galaxy in the reference system related to the Earth, then, according to Hubble and the Copernicus principle we can write:

$$\vec{r}(t) = a(t)\vec{r}(t_0) \quad (a \geq 0),$$

where $t_0$ is the present time and $a(t)$ is the scale factor depending only on time $t$, so $a = a(t)$. Thus $a(t = t_0) = 1$.

Taking into regard the uniformity of the universe the above expression is valid for any point of observation of the universe. Therefore, it is possible to define a universal cosmic time $t$, independent of the point of observation and assume that:

$$a(t = 0) = 0.$$  \hfill (2)

It means that for $t = 0$ the distances between all the galaxies of the universe become zero.

It can be easily shown that eq. (2) implies the Hubble law

$$v_{0r} = H_0 r_0,$$  \hfill (3)

where $v_{0r}$ is the present radial velocity of a given galaxy, $r_0 = |\vec{r}_0|$ is the distance to this galaxy and $H_0 = \frac{da}{dt} |_{t=t_0}$ is the so-called Hubble constant.
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From the point of view of the general theory of relativity the possibility of defining the universal cosmic time \( t \) for the whole universe is perplexing as, according to this theory time is relative and depends on the movement and intensity of the gravitation field, and the universe is full of movements and gravitation. Despite the omnipresent movement, all astronomical objects generate gravitation field and some drastically curve the spacetime. According to the theory of relativity the cosmos full of randomly distributed matter undergoing chaotic movements would not have one history because it would not have a universal cosmic time. However, in the largest scale the universe is not chaotic but uniform and isotropic. These very properties allow defining a cosmic time \( t \) describing the evolution of the observable universe. At the present stage of its evolution the following inequalities hold:

\[
\frac{da}{dt} \bigg|_{t=t_0} > 0
\]

and

\[
\frac{d^2a}{dt^2} \bigg|_{t=t_0} > 0
\]

which means that the galaxies move away from one another at a growing speed. Definition of the universal cosmic time permits a simple calculation of the current value of the volume of the observable part of the universe, known as the Hubble volume

\[
V_H = \frac{4}{3} \pi c^3 t_0^3,
\]

where \( c \) is the speed of light in vacuum.

As follows from the astronomical observations, the density of the universe energy being a sum of the densities of energies of the luminous matter, dark matter, false vacuum at present time \( \varepsilon (t = t_0) \) meets the condition:

\[
\varepsilon (t_0) > 0, \ \varepsilon = \text{constant} \ (\vec{r}).
\]

However, according to the calculations

\[
\varepsilon (t = 0) = \infty.
\]
According to the definition the open universe has infinite volume

\[ V = \text{constant} \ (t) = \infty, \ V/V_H = \infty, \]  

so its total energy in the matter and false vacuum meets the relation

\[ E = \text{constant} \ (t) = V \varepsilon \ (t) = \infty. \]  

It is very hard to imagine a physical process leading to generation of infinite energy. Taking into account this reason many cosmologists believe that the universe is not open but close so of a finite volume \( V < \infty. \)

According the calculations [1], for the close universe \( V \sim a^3 \) and therefore

\[ V = V \ (t), \ V_H/V \ll 1, \]
\[ V \ (t = 0) = 0, \ V \ (t) \varepsilon \ (t) < \infty. \]  

It means that at \( t = 0, \) when the distances between the galaxies become zero, also the volume of the closed universe must be zero. This implies that at \( t \approx 0 \) the volume of the closed universe could be much smaller than the volume of e.g. a proton. The objects of so small size are described not in terms of classical but quantum physics. The description of the closed universe in terms of the quantum theory is known as the quantum cosmology. Some aspects of the physics of time following from the assumptions of the quantum cosmology are considered below.

The energy of a closed universe is not only finite as follows from eq. (11), but taking into account the energy of the gravitation field its total energy \( E \) is exactly zero – as follows from the general theory of relativity (see [1]). This means that for a closed universe

\[ E = \text{constant} \ (t) = 0. \]  

This fact permits interpretation of the creation of the closed universe as a quantum fluctuation appearing spontaneously, causelessly and with no input of energy.
3 State evolution in a quantum closed universe

Quantum cosmology is based on the assumption that a closed universe can be assigned with a vector of state $|\Psi\rangle$ belonging to a certain space of states $\mathcal{H}$. It implies that in quantum cosmology the whole closed universe is treated as a stationary quantum system. According to the rules of the quantum theory, the vector of state $|\Psi\rangle$ should satisfy the linear equation in the sense of the superposition of states. This assumption should not raise any reservations for the early closed universe of the volume $V$, comparable with that of a single atom. For such an early universe the classical theory of gravitation so the general theory of relativity – is also valid as it could start existing only at the cosmic time $t$ much longer than the Planck time $t \gg t_p \cong 5.4 \cdot 10^{-44}$ s.

Because of the fact that the total energy of the closed universe is zero (12), its Hamiltonian has only one eigenvalue of zero, so

$$\hat{H} |\Psi\rangle = 0,$$ (13)

where $\hat{H}$ is a linear operator of total energy (Hamiltonian) of the closed universe.

Therefore, the linear unitary operator of the closed universe evolution $\hat{U}(t)$, defined by the expression

$$|\Psi(t)\rangle = \hat{U}(t) |\Psi(t = 0)\rangle$$ (14)

is a unit operator so $\hat{U}(t) = 1$, where $t$ is the universal cosmic time.

The above implies that the unitary evolution $U$, defined by the operator $\hat{U}(t)$, in the closed universe is frozen. In such a universe there are no changes, so it is impossible to define the parameter $t$ (cosmic time) numbering the states of the universe. This conclusion is obviously contradictory to the experience, which suggests that the assumptions of the quantum cosmology may be incorrect. However, it does not have to be so because apart from the unitary evolution of quantum systems, the quantum theory predicts the nonunitary jumpwise and irreversible evolution, sometimes denoted by $R$, which appears at the moment of a quantum measurement.

In view of the above, if we wish to conserve the assumptions of quantum cosmology, we have to assume further that the changes
observed in the universe are generated by the evolution $R$. This means that the evolution $R$ cannot be reduced to the unitary evolution $U$, as has been postulated by many hitherto proposed interpretations of the quantum theory [2]. Consequently, the evolution $R$ can be generated only by an element of reality indescribable by eq. (13), so this element cannot be subjected to the quantum physics laws. This would further imply that the quantum physics and quantum cosmology do not make a closed system of reasoning.

From the set of a few interpretations satisfying the demands of quantum cosmology, those assuming nontrivial evolution $U$ should be disregarded. These are: the Everett’s interpretation of many worlds and its generalisation based on the phenomenon of decoherence, the Gell-Mann & Hartle interpretation of the consistent histories [2] and Cramer’s transaction interpretation [3]. The main question arises how to explain changes taking place in the universe under the assumptions taken.

Because of enormous complexity of the universe it is fully justified to assume that the eigenstate of the universe Hamiltonian $\hat{H}$ is strongly degenerated, so eq. (13) is satisfied by a huge but countable – because of a finite volume of the universe – number of state vectors $|\Psi_j\rangle$, where $j = 1, 2, \ldots, |\Psi_j\rangle \in \mathcal{H}$.

Physically distinguishable are only the linearly independent states $|\varphi_k\rangle$, belonging to the space of states $\mathcal{H}$, where $k = 1, 2, \ldots$. After orthonormalisation they satisfy the condition

$$\langle \varphi_k | \varphi_{k'} \rangle = \delta_{k,k'}$$

and the condition of completeness

$$\sum_k |\varphi_k\rangle \langle \varphi_k| = \hat{1},$$

where the summation goes over all $k$ values. Therefore an arbitrary state of the universe $|\Psi\rangle$ can be expressed as a linear superposition of the states $|\varphi_k\rangle$

$$|\Psi\rangle = \sum_k \langle \varphi_k | \Psi \rangle |\varphi_k\rangle.$$  

It is worth noting that at least because of (11) there is no apparatus in the universe enabling performance of an ideal operation $R$ for the
whole universe but only such that would enable performance of partial operations $R$ related to non-ideal quantum measurements. The non-ideal measurement gives only partial knowledge of the state of the system. The knowledge and operation $R$ require a self-conscious subject not subjected to linear laws of quantum physics (in the sense of the state superposition principle) as it was for the first time noted in 1929 r. by Charles G. Darwin [4].

The self-conscious subject could not have appeared in the universe being in the state of absolute calmness to perform a quantum measurement generating operation $R$. For this reason we have to assume that it is not that the subject exists in the universe but the universe exists in the minds of the subjects. We know that there are many human subjects believing that they exist in one universe, therefore, we have to assume that there is a factor not subjected to the linear laws of quantum physics which generates in a correlated way the impressions of existence in one universe in the minds of the human subjects. This assumption leads to the ontology known as the objective idealism. Apart from the above arguments this ontology is supported by the fact that on its grounds it is possible to derive the fundamentals of the mathematical formalism of the quantum theory [5],[6].

Because of the inequality (11) no human subject is able to determine the full state of the universe $|\Psi\rangle$ but is only capable of determining the state of a very small fragment of the universe denoted by $A$. The other unobservable part of the closed universe is denoted by $B$. In order to determine the maximum obtainable information on the state of the subsystem $A$, we have to apply the known method of reduced density operators [7], used among others in quantum thermodynamics [8].

The reduced operator of density of the observable part of the universe is defined by the expression [8]

$$\hat{d}_A = \text{Tr}_B [\hat{d}] , \quad (18)$$

where $\hat{d}$ is the operator of the universe density equal to

$$\hat{d} = |\Psi\rangle \langle \Psi| , \quad (19)$$

$\text{Tr}_B[...]$ is the partial trace over the states of the unobservable part of the universe $B$.\n
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Using the operator $\hat{d}_A$ we can calculate the expected value of an arbitrary observable $p$ characterising the observable part of the universe $A$. Having assigned the self-adjoint operator $\hat{p} = \hat{p}^+$ to the observable $p$, we can write its eigenequation in the form:

$$\hat{p} |p\rangle = p |p\rangle,$$

where

$$\langle p | p' \rangle = \delta_{p,p'},$$

and the condition of completeness

$$\sum_p |p\rangle \langle p| = \hat{1}_p,$$

$\hat{1}_p$ – is the unit operator acting in the space of $\mathcal{H}_A$ states of the subsystem $A$.

The expected value of the observable $p$ is defined as

$$\langle \hat{p} \rangle = Tr_A \left[ \hat{p} \hat{d}_A \right],$$

where $Tr_A[...]$ is the partial trace over the states of the subsystem $A$.

The reduced operator of density $\hat{d}_A$ defined by eq. (18) can be expressed in the basis of eigenvectors $|p\rangle$ of the operator $\hat{p}$, using the condition of completeness (22),

$$\hat{d}_A = Tr_B \left[ \hat{d} \right] = \hat{1}_A Tr_B \left[ \hat{d} \right] \hat{1}_A = \sum_p \sum_{p'} |p\rangle d_{pp'} \langle p'|,$$

where

$$d_{pp'} = \langle p | Tr_B \left[ \hat{d} \right] | p' \rangle.$$

The ideal measurement of the observable $p$ generates the following evolution of $R$

$$\hat{d}_A = \sum_p \sum_{p'} |p\rangle d_{pp'} \langle p'| \xrightarrow{R} |p''\rangle \langle p''| = \hat{d}_{A''}.$$

It means that as a result of the measurement of the observable $p$ we get the eigenvalue of the operator $\hat{p}$ equal to $p''$. The procedure requires that the following condition is met

$$d_{pp'} \xrightarrow{R} \delta_{p,p''} \delta_{p',p''}.$$

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Using eq. (19) and (22), eq. (28) can be written as

\[ d_{pp'} = \sum_k \sum_{k'} \langle p | \text{Tr}_B [ |\varphi_k\rangle \langle \varphi_k | \Psi \rangle \langle \Psi | \varphi_{k'} \rangle \langle \varphi_{k'} | | p' \rangle \rangle. \]  

(28)

In this expression the states \( |\Psi\rangle \) and \( |\varphi_k\rangle \) are given. Satisfaction of (27) at the moment of the ideal measurement of the observable \( p \) can only limit the range of summation over the states \( |\varphi_k\rangle \). In this way each measurement of a state of any subsystem of the universe reduces the number of admissible states \( |\varphi_k\rangle \) in the superposition (17). In other words each measurement or observation performed by a self-conscious subject restricts the space \( \mathcal{H} \) of the states of the universe admissible by equation (13), so that

\[ \mathcal{H} \xrightarrow{R} \mathcal{H}^1 \subset \mathcal{H}, \]  

(29)

where \( \mathcal{H}^1 \) is the subspace of the possible states of the universe after the measurement.

The universe evolution can be symbolically presented as a sequence of the processes \( R_1, R_2, \ldots \) restricting the space of the universe states

\[ \mathcal{H} \supset R_1 \supset \mathcal{H}^1 \supset R_2 \supset \mathcal{H}^2 \supset R_3 \supset \mathcal{H}^3 \supset \ldots. \]  

(30)

The process of reduction of the space of the possible states of the universe is irreversible and follows from the irreversible state of knowledge of the subjects. It explains the well known fact of the irreversibility of the quantum measurement. In this approach this process is an essential reason for the irreversibility of the time flow in the universe.

Let’s note that the dimensionless von Neumann entropy \( S \) of the whole universe is constant and equal zero, so that

\[ S = -\text{Tr} \left[ \hat{d} \ln \hat{d} \right] = 0. \]  

(31)

On the other hand, the entropy of any subsystem of the universe, e.g. the observed part \( A \) interacting with the unobserved part \( B \), is different from zero and equal

\[ S_A = -\text{Tr}_A \left[ \hat{d}_A \ln \hat{d}_A \right] > 0 \]  

(32)
and can increase. It means that the universe as a whole cannot have any definite temperature assigned to it. Temperature can be defined only for not fully isolated subsystems of the universe being at the equilibrium or close to thermodynamic equilibrium [8].

A huge number of subjects continually perform an infinite number of observations and measurements, which continuously reduce the space of admissible states of the universe. Therefore, besides the change in the reduced density operator (26), generated by measurement of observable \( p \), there is a continual and irreversible change \( \hat{d}_A \) caused by other uncountable observations and measurements, that can be expressed as

\[
\hat{d}_A = \hat{d}_A(t),
\]

where parameter \( t \), numbering the irreversible changes in the state of the observable part of the universe \( A \), described by the reduced density operator \( \hat{d}_A \), can be identified as physical time.

Let’s consider the question in more detail. The self-conscious subject for obvious reasons must be a macroscopic system. This leads to a conclusion that the subjects can exist in a large enough universe that can be to a sufficient accuracy described by eq. (13) in the so-called quasi-classical approach.

As follows from some considerations within the general theory of relativity, the Hamiltonian of the universe \( \hat{H} \) can be expressed as a sum of two terms [9]

\[
\hat{H} = \hat{H}_g + \hat{H}_m,
\]

where \( \hat{H}_g \) is the operator of gravitational energy and \( \hat{H}_m \) is the operator of energy of the matter in the universe. A solution to eq. (13) is looked for in the quasi-classical approach, which assumes the following form of the state vector of the universe \( |\Psi\rangle \):

\[
|\Psi\rangle = e^{i \frac{S_g}{\hbar}} |\chi\rangle,
\]

where the term \( \exp(i \frac{S_g}{\hbar}) \) describes the fast-changing part of the vector of state, while \( |\chi\rangle \) describes its slow-changing part, and \( S_g \) is the gravitational action of the universe. Substituting (34) and (38) to (13) we get

\[
\left( \hat{H}_g + \hat{H}_m \right) e^{i \frac{S_g}{\hbar}} |\chi\rangle = 0.
\]
The condition of applicability of the quasi-classical approach is satisfaction of the inequality \( S_g \gg \hbar \), which is valid for a sufficiently large closed universe. Assuming that the operator \( \hat{H}_m \) does not act on \( S_g \), so
\[
\hat{H}_m |\Psi\rangle = e^{i \frac{S_g}{\hbar}} \hat{H}_m |\chi\rangle
\] (37)
and making use of the form of \( \hat{H}_g \) obtained by canonical quantisation of the Hamilton function of a closed and uniform universe following from the general theory of relativity, (36) can be expressed in a symbolic form [9]
\[
\hbar^0 (\text{left side of Hamilton-Jacobi equation}) |\Psi\rangle + \left( \hat{H}_m |\chi\rangle - i\hbar \frac{d|\chi\rangle}{d\tau} \right) e^{i \frac{S_g}{\hbar}} + \hbar^2 (0) = 0
\] (38)
Equalling the terms of the Planck constant \( \hbar \) small relative to \( S_g \), in the same power (the term \( \hat{H}_m |\chi\rangle \) is included into the equation with \( \hbar \) in the first power and the term with \( \hbar^2 \) is neglected), we get a set of two equations: the Hamilton - Jacobi equation for the gravitational interactions of the universe and the equation
\[
i\hbar \frac{d|\chi\rangle}{d\tau} = \hat{H}_m |\chi\rangle ,
\] (39)
where \( \tau = \tau(a) \) is a function of the scale factor defined by eq. (1) only. Therefore, parameter \( \tau \) is the same at each point of the universe. The scale factor \( a \) changes as a result of a reduction of the space of states \( \mathcal{H} \) of the universe caused by a huge number of measurements and observations of the self-conscious subjects.

If the continuous parameter \( \tau \) is identified with the cosmic time \( t \), then the equation
\[
i\hbar \frac{d|\chi\rangle}{dt} = \hat{H}_m |\chi\rangle
\] (40)
becomes identical to the Schrödinger equation describing the evolution of matter in the universe. From this point of view the Schrödinger equation is approximate. Eq. (40) can be treated as a quasi-classical approximation of eq. (13). The exact (13) does not contain time, which appears as \( t \equiv \tau \) in the quasi-classical approach in which the gravitational function of the universe interaction \( S_g \gg \hbar \). For this
reason there is no operator of time in the quantum theory, so time is 
not an observable but only a parameter, which is a very important 
conclusion for physics, following from the quantum cosmology.

Eq. (16), (40) and (43) give
$$|\Psi\rangle = e^{i\frac{S_B}{\hbar}} e^{-i\hat{H}_m t/\hbar} |\chi(0)\rangle,$$  
where $|\chi(0)\rangle = |\chi(t = 0)\rangle$. Substituting (41) to (9) we get an approximate form of the density operator of the universe
$$\hat{d}(t) \cong e^{-i\hat{H}_m t/\hbar} |\chi(0)\rangle \langle \chi(0)| e^{i\hat{H}_m t/\hbar}.$$  

In this way, in the quasi-classical approach a unitary evolution $U$ appears. Substituting (41) into 18 we arrive at the explicite function (33) describing the time dependence of the reduced density operator
$$\hat{d}_A(t) = \text{Tr}_B \left[ e^{-i\hat{H}_m t/\hbar} |\chi(0)\rangle \langle \chi(0)| e^{i\hat{H}_m t/\hbar} \right].$$  

Because of the approximate character of the unitary evolution $U$ it is not irreversible, in contrast to the primary process described by eq. (30), because eq. (40) describing the process $U$ is invariant with respect to the transformation $t \rightarrow -t$.

It is easy to show that the operator $\hat{d}_A(t)$ permits calculation of the probability $P$ of getting the eigenvalue $p''$ of the operator $\hat{p} = \hat{p}^+$ in the measurements of the observable $p$, if prior to the measurement the subsystem $A$ was in the state described by the reduced density operator $\hat{d}_A(t)$. This probability is
$$P \left( \hat{d}_A(t) \rightarrow \hat{d}_A'' \right) = \text{Tr}_A \left[ \hat{d}_A(t) |p''\rangle \langle p''| \right],$$  
where $\hat{d}_A''$ is defined by eq. (37).

Eq. (43) and (44) together with (23) imply that in the macroscopic universe the evolution of any arbitrarily selected part of the universe is subjected to certain rules permitting predictions of the probability of this or another physical event.

4 Concluding remarks

Our considerations of the physics of time are based on the assumption that we live in a closed universe (of finite volume) that we
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can describe by the best theories we have that is the general theory of relativity and the quantum theory. Moreover, on the basis of the astronomical observations we assume that in a large scale the universe is uniform and isotropic. These assumptions are very well justified at the present stage of development of physics and cosmology. For this reason they are also a good starting point of the considerations of the properties of physical time.

The general theory of relativity admits the existence of closed universe. According to this theory the total energy of such a universe is zero. The theory permits construction of a Hamilton function of such a universe and on the basis of this function – the relevant energy operator (Hamiltonian). The Hamiltonian is fundamental for analysis of the quantum properties of the uniform and closed universe.

According to the rules of the quantum theory, the Hamiltonian of the universe whose total energy is zero has only one eigenvalue equal zero. The spectrum of the operator has only one numerical value – zero. The eigenstate of the universe corresponding to this eigenvalue is enormously degenerated because of the high complexity of the universe. This fact imposes certain restrictions on the space of the university states. The above considerations imply a few very interesting conclusions for the physics of time in a closed universe.

1. Strictly speaking a closed universe is not subject to a unitary evolution $U$. For the closed universe the operator of evolution is identical to the unit operator. Therefore, the changes observed in the closed universe can be generated only by a non-unitary, irreversible and jumpwise evolution $R$, related to the quantum measurement.

2. All attempts at reducing the evolution $R$ to the unitary evolution $U$ are bound to fail. This means that no process of decoherence can explain the reduction of the superposition of states at the moment of quantum measurement, which implies falsification of a number of interpretations of the quantum theory. The process of quantum measurement and the related evolution $R$ cannot be generated by the elements of reality subjected to the linear laws of the quantum theory. Therefore, the quantum theory does not make a closed system of reasoning.
3. After Charles G. Darwin [4] and John von Neumann we assume that the element of reality not subjected to the laws of the quantum theory and generating the evolution $R$ is the awareness of being of the subjects. However, in a closed universe in which no changes could occur also no subject could appear. Therefore, we have to assume the idealistic point of view according to which we cannot say that the subjects exist in the universe but that the universe is spanned in the minds of the subjects. The subjects have an impression of existing in one universe because individual universes generated in their minds are mutually correlated by a factor transcendental with respect to them. Consideration of the nature of this factor (see [5]) is for obvious reasons beyond the range of interest of the physics being the subject of this paper.

4. All the above leads to a conclusion that prior to the appearance of the first man (assuming that we are alone in the universe) the universe was in the invariable quantum state being a superposition of the quantum states corresponding to many possible universes. Only at the moment of appearance of the first man, in a jumpwise manner the universe was actuated with the laws of physics and the past allowing the existence of man (the anthropic universe appeared), from all potentially existing universes.

5. From this point of view, the self-conscious subjects feel the lapse of time because their knowledge recorded in their minds thanks to the memory undergoes continuous changes. The changes are to a high degree regular and are correlated for all subjects so the notion of time appears to them in a natural way (as a parameter numbering the states of the minds) sensed mainly as changes in the states of the universe and their bodies. Hence, the psychological and physical time have the same source. As the change in the knowledge is irreversible, also the physical time is, which is generated by the evolution $R$. Only the unitary evolution $U$ of chances for a given change in the subject’s knowledge (represented by the state vectors or reduced density operators) is reversible. The unitary evolution $U$ is only an approximate (quasi-classical) description of the universe evolution.
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[10],[11] whose origins are the sequences of processes R (see eq. (21)).

References


