"Do not fear to be eccentric in opinion, for every opinion now accepted was once eccentric"

Bertrand Russell
TOWARDS DETERMINATION OF THE
LAPLACE GRAVITY PARAMETER $h$

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Abstract

We review a compressive model for material inertia and the
effect of gravitation which follows naturally from it. We show
that this model is consistent with the relativity principle, and
suggest how a slight departure in the observed advance of the
perihelion of the orbit of Mercury could put an upper limit on
the Laplace screening parameter $h$. 
1 Introduction

It can be argued that relevant sections in the Physics of Aristotle (∼350 BCE) – particularly Book 7 – point to an intuitive understanding of the cause of material inertia in terms of the effect of pressure from a matter-surrounding medium. Gravitation would be a natural consequence of such an effect (cf. Budding, 2005, 2007). Such concepts are often referred to in connection with the thesis of Le Sage (1784), that was updated and re-addressed in the series of papers in the book Pushing Gravity (Edwards, 2002). An impression can arise, however (e.g. van Flandern, 2002; Schilling, 2004), that such older, intuitive models are at variance with the formalized discussion of Einstein in the general theory of relativity (GTR — cf. also Eddington, 1920). But it is not difficult to show – on general physical arguments – that this is not necessarily the case. Indeed, the Schwarzschild metric for a stationary ‘attracting’ point mass follows naturally in its normal form if the velocity of gravity-mediating particles (‘gravitons’) is the same as that of electromagnetic radiation – at least to the zeroth order in the Laplace absorption parameter $h$ for gravitational force.

The next section reviews this point. A short third section considers the well-known practical test of the advance of perihelion of Mercury. This could provide an upper limit to the value of the $h$, but that result appears unlikely to be able to give an actual determination for some considerable time yet.

2 Inertial compression and relativity

A simplified physical picture can be produced in which the material inertia of moving objects is discussed along lines typically given for (special) relativity. Consider first the situation in the $y$-direction for a small object moving with constant velocity $v$ parallel to a given $x$-direction (Fig. 1). For convenience, this elementary object is taken to be of rectangular shape, with sides parallel to the co-ordinate axes. The compressive model for inertia posits a uniform and isotropic surrounding field of particles (‘gravitons’) energized to an energy density $U$, that penetrate the object, the vast majority of which emerge without any significant material interaction. However, the absorption of some small proportion of the energy of the field produces a compression of the object, proportional, and scalably equivalent, to its inertia $I_u$ that is its reluctance to acceleration (cf. Budding, 2005).
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![Diagram](image)

Figure 1: Linear motion in $x$ direction (schematic). The arrowed rays incident on the moving object of length $x_1$ in the reference frame are seen as normally incident in the moving frame.

In the ‘rest’ frame of reference, it would seem that there is no difference between the compression in the $y$-direction experienced by a moving object and a similar object that is at rest. Any graviton missed at the back of the object as a result of the forward motion is effectively made up for by another one further forward that would have been missed if the object had not been moving.

In the moving frame, however, any graviton perceived to be coming directly from the positive $y$-direction must actually be coming with a component of its (constant) speed $c$ (in the rest frame) in the positive $x$-direction equal to that of the object (Fig. 1). Hence its impact momentum in the $y$-direction is reduced in the ratio $\sqrt{1 - \beta^2}$, where $\beta = v/c$. But this applies to all microscopic interaction components in the $y$-direction of the moving frame mediated by $c$-speed interactions, including those in a reference ‘clock’ moving with the object (cf. Larmor, 1897). The main function of such an idealized (proper) clock is that it produces a repetitive reference motion essentially orthogonal to the changes of position being timed (those in the $x$-direction). A conceptual pendulum, say, moving in the $yz$ plane corresponds to such a clock. In effect, the interval between swings for a pendulum, powered by $y$-direction graviton interactions, would
slow down, so that the same amount of graviton momentum transfer would occur in the same measured time interval. The same net compressive y-force would in this way be independently measured in both frames.

The situation in the $x$ direction can be considered in terms of the communication of microscopic interactions of elastic particles, of which it is taken to be composed, set in motion from one $yz$ section to its particular neighbour. This model is minimalistic: we do not enquire into the detailed nature of the particles of the body, only that their motion is rapidly communicated from one to another and that they have some response to the graviton field. Conservation of momentum continues in the same way as for this same object when nearly at rest in the rest frame only when the neighbouring $yz$ sections are compressed in the $x$-direction in the same ratio $\sqrt{1 - \beta^2}$ as above. Newton’s second law holds for such a model of material motion started from rest (Budding, 2005), and from that we deduce that an impulse large enough to propel an object forward with discernible velocity $v$ has a potential energy of compression

$$E = k(x_1^2 - x_2^2)/2, \quad (1)$$

where $x_1$ is the natural length in the rest frame and $x_2$ is that in the moving frame. The full energy of the motion is made up of the potential energy of compression as well as the kinetic energy of the impulse. Again from the law of motion, it follows that the modulus $k$ scales in terms of the compression in the reference frame, i.e. $k = I u_c^2 / x_1^2$, where $c$ is the speed of the impulsive wave through the object. Since the potential energy equals the kinetic energy for this motion, we must have

$$v^2 = \frac{c^2}{x_1^2} (x_1^2 - x_2^2). \quad (2)$$

Equation (2) expresses the well-known contraction of moving objects (Fitzgerald, 1889). The compression implies an increased overall inertia of the moving object in the same proportion as its deceased length. The measured velocity of gravitons in the moving frame turns out the same in both $x$ and $y$ directions, and still the same as in the rest frame, thus confirming the principle of relativity (Einstein, 1920;
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with reference to the potential energy of compression, see also Fayngold, 2008).

At the level of individual material particles, their inertia is measured by any impulse they receive $\Delta J$, divided by the square root of the product of their cross-section and rigidity $\sqrt{\sigma \kappa}$. The former measures an interaction length, which, from the foregoing, contracts in the direction of motion, thereby increasing the inertia of the material particle. Any period of response of such particles to an impulse of given mean magnitude $\Delta J$ is scaled by $I/\sqrt{\kappa}$: the moving particle becoming more ‘inert’, its response time correspondingly dilated. Although the general compression of objects is regarded as coming from the supposed cosmic field as the principal source of local inertia, conservation of momentum among individual particles is somewhat more general, and $I$ can take a more basic meaning as the mechanical mass $M$ of the particle. It can be countenanced, though, that if the contraction approaches the scale of the material particle’s interaction length itself, the particle may reach some microscopic equivalent of a ‘yield point’, beyond which there is increasing difficulty in holding itself together. The limiting case of $x_2 = 0$ implies the potential energy would rise to $\frac{1}{2}Mc^2$ from (1), so, for the total energy of such a particle, we find Einstein’s famous expression

$$E_{\text{tot}} = Mc^2.$$ (3)

Although the discussion so far refers directly to special relativity (constant uniform velocities), the arguments extend to the general case. Since, following Eqn (2), uniform linear motion is equivalent to a compression in the direction of motion, acceleration, or the rate of change of velocity (of a body), corresponds to a rate of change of this compression.

The relationship of gravitation to acceleration in GTR is associated with an idea of Mach, i.e. that physical laws are ultimately derive from clearly recorded perceptions of events by systematic observers. A passenger on a (horizontally) cornering vehicle feels a force pushing him away from the centre of turning. At the time when his relative movement is towards the side of the vehicle, immediate perceptions suggest only this ‘centrifugal’ effect. Perhaps later, when still, but pushed against the side of the vehicle, he may think that it was the car that turned, while his body’s inertia kept him in the direction…
he took to be outwards. At this point, he is in equilibrium relative to the car: the acceleration from his inertia relative to the moving frame now balanced by the ‘centripetal’ inward push of the vehicle wall. Could the idea of ‘falling’ be due to an upward accelerating surface of the Earth, and our coming to rest against this a comparable balance of an imposed acceleration and an inertial tendency?

A key point is that we are regarding inertia as proportional to a general compression. The free movement of an inert body would then only be uniform and linear in a region where the universal compressive field is isotropic and uniform. Near a very massive body, such as the Earth, this is not the case. A compressive strain is thus set up across a body of finite vertical size, with a consequent proportional differential stress. The vertical integral of this stress differential corresponds to a net mechanical action on the body. It then accelerates, as a consequence of a differential loss of inertia (a relatively very slight loss of compression) in the downward direction. Relative to the falling observer the Earth is, of course, accelerating upwards; but as with the cornering car passenger, it seems much more natural to regard the observer as moving relative to fixed surroundings. Structural stability of the Earth implies that objects on its surface equilibrate. The imposed acceleration, analogous to the cornering car’s wall, now associates with the restoring upthrust that maintains this equilibrium against the inertial tendency to keep falling.

If a falling object continues to very high speeds it will again increase its overall inertia according to Eqn (2). Continued acceleration towards a centre of gravitational attraction would then tend to depart from a simple Newtonian prescription. Since that acceleration is expressed as a rate of changing distances in successive time intervals, and depends almost entirely only on the mass of the attracting body, one may expect an appropriate form for the modified acceleration to be devisable in terms of appropriate modifications to space and time co-ordinates to be applied to any falling body. The setting up of such modifications for a manifold of space and time pervaded by a gravitational field is a main objective within GTR.

The principle of relativity referred to above can be dealt with in the setting of changing velocities with the aid of the Minkowsky construction for differential elements of space and time together, i.e.
as in a 4-dimensional ‘space-time’ continuum. This is written as

\[ ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 . \]  

(4)

Eqn (4) implies that events observed to be separated by \( dx, dy, dz \) in space and \( dt \) in time, that actually occur simultaneously, have been properly timed when the increment \( ds \) is zero. A special meaning attaches to the quantity \( c \), that remains constant for moving observers. This propagation velocity, introduced above in connection with the apparent linear motion of material bodies, represents the maximum velocity with which events can be communicated from one particle to another. It is well known that this is generally identified with the velocity of electromagnetic radiation: the ‘speed of light’. The form of the Minkowsky construction, by setting the second power of \( ds^2 = 0 \) for a light beam travelling between two points with constant locally measured velocity \( c \), also implies that its propagation follows the shortest path between two locations.

Let us consider the gravitational field around an idealized, point-like mass. Symmetry suggests the appropriateness of spherical polar co-ordinates centred on the mass-point, and at distances so large that the cosmic field becomes uniform and isotropic, Eqn (4) will take the form

\[ ds^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 . \]  

(5)

More generally, however, we should expect a form such as

\[ ds^2 = \sum_{\alpha\beta} g_{\alpha\beta} x_{\alpha} x_{\beta} + ... \]  

(etc.),

where we have replaced the familiar \( x, y, z \) and \( ct \) co-ordinates in (4) by the standardized \( x_1, x_2, x_3, x_4 \), and \( g_{\alpha\beta} \) make up the metric coefficients of the co-ordinate transformation. Dimensional arguments, stemming particularly from the coefficient of the second-order time increment, indicate that the \( g \)’s have the physical character of generalized force potentials.

A ‘law of gravitation’ can be understood as a relationship affecting the \( g \)’s, that would hold for values of such coefficients observed in practice. Since these \( g \)’s define any given system of co-ordinates, the
implication is that a gravitation law would connect systems of co-ordinates applicable to real cases. As and when new co-ordinates become relevant, there obtain corresponding values of the $g$’s, where the differential equations connecting the new $g$’s and new co-ordinates continue *covariantly* from what held between the old $g$’s and old co-ordinates.

It can be shown (cf. Eddington, 1920) that very general conditions for a source-free gravitational field are equivalent to the vanishing of the fourth rank Riemann-Christoffel tensor (large to write out in full), but this is reduced, in Einstein’s formulation, to the second rank symmetric tensor equation

$$G_{\sigma\tau} = 0 ,$$  \hspace{1cm} (7)

the indices $\sigma$ and $\tau$ running from 1 to 4. This is the four-dimensional counterpart to the Laplace equation for a classical field. The 10 separate conditions thus expressed provide the basis of the GTR formulation of gravitation. Any set of values of the 10 independent metric coefficients $g_{\alpha\beta}$ that satisfy (7) will correspond to a possible set of co-ordinates applying to a source-free field.

These reduced Riemann-Christoffel conditions can be spelled out as

$$G_{\sigma\tau} = -\frac{\partial}{\partial x\alpha} \{\sigma\tau, \alpha\} + \{\sigma\alpha, \beta\} \{\tau\beta, \alpha\} + \frac{\partial^2}{\partial x_s \partial x_t} \log \sqrt{-g} +$$

$$-\{\sigma\tau, \alpha\} \frac{\partial}{\partial x\alpha} \log \sqrt{-g} ,$$  \hspace{1cm} (8)

where $\{\alpha\beta, \gamma\}$ represents the Christoffel 3-index symbol appearing in the covariant differentiation of the $g_{\alpha\beta}$ tensor, defined as $\{\alpha\beta, \gamma\} = \frac{1}{2} g^{\gamma\epsilon}(\partial g_{\alpha\epsilon}/\partial x_\beta + \partial g_{\beta\epsilon}/\partial x_\alpha - \partial g_{\alpha\beta}/\partial x_\epsilon)$, and the right hand side is summed in the repeated index $\epsilon$ (Einstein index convention).

Symmetry and other simplifications applying to the elementary situation considered permit

$$ds^2 = -e^\lambda dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + e^\nu dt^2$$  \hspace{1cm} (9)

as an exploratory form, whose $g$’s may satisfy (8), where $\lambda$ and $\nu$ are arbitrary functions of $r$. We thus have, $g_{11} = e^\lambda$, $g_{22} = -r^2$, $g_{33} = -r^2 \sin^2 \theta$, $g_{44} = e^\nu$, and $g_{\sigma\tau} = 0$, when $\sigma \neq \tau$. The co-ordinate
system remains orthogonal, but there appear scaling factors in the radial and time dimensions relative to a given spatial extension in the angular co-ordinates. The determinant $g$ of the $g$-coefficients reduces to its leading diagonal, so that $-g = e^{\lambda + \nu r^4} \sin^2 \theta$, and its cofactors $g^{\sigma \sigma} = 1/g_{\sigma \sigma}$. This allows considerable simplification of the general expression (8) — only 9 of the 40 possible independent Christoffel symbols are non-zero, for example — and, after some manipulation, it is found that the conditions (8) require the trial form (9) to reduce to

$$ds^2 = -\gamma^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 + \gamma dt^2 ,$$

with $\gamma = 1 - b/r$, where $b$ is a constant of integration. The resemblance of $\gamma$ to a gravitational potential will become clearer with an appropriate physical interpretation of $b$.

We can show that (10) is consistent with a generalization of the transforms of special relativitiy by setting $x$ in (2) to be in a given radial direction of acceleration $r$. We have, for two different positions along the path, $r_1$ and $r_2$, say, (and these positions can be regarded independently of time),

$$\delta r_2^2 - \delta r_1^2 = \delta r_1^2 \frac{2E}{I_u c^2} ,$$

or

$$\delta r_2^2 = \delta r_1^2 (1 + \frac{2E}{I_u c^2}) ,$$

where $E$ is the increase in potential energy of a point mass moving in the positive direction from $r_1$ to $r_2$. In the simple limiting case considered, one single large ‘attracting mass’ $M$ at the fixed origin is regarded as the source of changes of potential for the moving particle, so that the second item in the parentheses in (12) can be replaced by the conventional form $-2GM/rc^2$. The natural length $\delta r_1$ in a local frame of reference is equivalent to a proper time $\delta s_r/c$, while $\delta r_2$ gives the corresponding variation of this length applying to the purely spatial component of radial separation $r$, so that, generally,

$$\delta r^2 = -\delta s_r^2 (1 - \frac{2GM}{rc^2}) ,$$

or

$$\delta s_r^2 = -\delta r^2/(1 - \frac{2GM}{rc^2}) .$$
Relative changes of material compression are also associated with changes in the local elapsing of measured time intervals $\delta t_1$, compared with intervals $\delta t_2$, at some other position, as argued before. In the present situation, these changes relate to variations of radial separation from the centre of attraction, and are in exact proportion to the compression, so that

$$c^2 \delta t^2_2 = c^2 \delta t^2_1 / \left(1 - \frac{2GM}{rc^2}\right).$$

(15)

We can write again, in the manner of (9)

$$\delta s^2 = c^2 \delta t^2 \left(1 - \frac{2GM}{rc^2}\right).$$

(16)

Taking into account both the spatial contraction and time dilation effects from (14) and (16), we can rewrite (6) for the situation at distance $r$ from the mass centre $\mathcal{M}$ as

$$ds^2 = c^2 \left(1 - \frac{2GM}{rc^2}\right) dt^2 - dr^2 / \left(1 - \frac{2GM}{rc^2}\right) - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2.$$

(17)

It is noteworthy that $b$, that enters into (10) through $\gamma$, is equivalent to $2GM/c^2$ in (17). The law of gravitation, expressed through Einstein’s field equation (7), thus supports the expression for the potential used in (13) and confirms conventional physical interpretation of gravitational potential energy in this context. Since there are no changes of angular momentum in this model, there are no effects on $ds^2$ associated with differential changes in the increments $d\theta$ and $d\phi$.

Eqn(17) expresses the well-known Schwarzschild metric (Schwarzschild, 1915), that figures in most practical tests of GTR. It can be seen from this discussion that nowhere does a compressive model of inertia and gravitation, in the zeroth order in $h$, given as Equations (9) and (16) in Budding (2005), depart from standard relativity theory in its main testable findings when graviton velocity is the same as that of light.

3 Practical test

What would be the effect on the Schwarzschild metric for an accelerated object of finite size? We now note the consequence of the
compression theory that gravitational attraction (weight) is slightly inefficient as an accelerative effect when converted into a series of momentum changes throughout a body having appreciable matter content (Budding, 2005).

The absence of angular momentum variation in Schwarzschild (central mass) attractions means that one of the angular variables, θ say, can be eliminated: orbital motion is in the ‘equatorial’ plane θ = π/2 with no rotation about a horizontal axis. The constant angular momentum vector about the axis θ = 0 and constancy of observed effects in the proper frame (the relativity principle) similarly allows the integral

\[ r^2 \frac{d\phi}{ds} = \eta , \tag{18} \]

whereupon (10) can be arranged as

\[ \gamma^{-1} \frac{dr}{ds}^2 + r^2 \frac{d\phi^2}{ds} = c^2 \gamma \frac{dt^2}{ds} - 1 , \tag{19} \]

This can be further tailored to something like the normal expression for a Keplerian orbit by multiplying the left side by γ and moving a small part of the term in the rate of angular variation over to the right side, thus

\[
\left( \frac{dr}{ds} \right)^2 + \left( \frac{r^2 d\phi}{ds} \right)^2 = \gamma \left[ c^2 \gamma \left( \frac{dt}{ds} \right)^2 - 1 \right] + (1 - \gamma) \frac{\eta^2}{r^2} . \tag{20}
\]

The left side of (20) now looks like the regular kinetic term \( v^2 \), but we should divide by \( c^2 \), if regarding \( s \) as measuring proper time. From (16), the coefficient of \( (dt/ds)^2 \) in the square parentheses would seem to cancel out, but there is an element of arbitrariness about the selection of the constant position, applying to the time intervals \( \delta t_0 \), to which the orbit is referred. This involves again the same factor given in (2), so that we end up on the right side with

\[ \left[ \gamma \left( \frac{1}{1 - v_0^2 / c^2} \right) - 1 \right] + (1 - \gamma) \frac{\eta^2}{r^2} . \]

If we write \( v_0^2 = GM/a \) in this, insert the full expression for \( \gamma \) and ignore higher orders than the first in small terms, (20) now reduces...
to
\[ v^2 = G\mathcal{M} \left[ \left( \frac{2}{r} - \frac{1}{a} \right) + \frac{2\eta^2}{r^3 c^2} \right], \]  
(21)
which looks like the familiar Keplerian form for the orbital velocity, except for the small additional term involving the angular momentum constant \( \eta \) on the right.

It can be shown (cf. e.g. Eddington, 1920) that this small extra term leads, for an elliptic orbit of eccentricity \( e \), to a slow rotation of the orbit as a whole (advance of perihelion) which, per orbit, amounts to
\[ \Delta \phi = \frac{3\eta^2}{a^2 (1 - e^2)}. \]  
(22)
\( \Delta \phi \) is measured in the combination \( e \Delta \phi \), which for the planet Mercury amounts to some 8.82 arcsec per century. The measured value falls short of this by 0.07 arcseconds, however, i.e. some 0.8% of its value (Shapiro et al., 1972), although this is within the standard error of the measurement.\(^1\) This can be understood physically as the fall of potential energy for unit amount of matter \( E/I_u \) in (12) not being quite up to the same level as the substituted form \( -G\mathcal{M}/r \) in (13), that would apply uniformly to all matter in the zeroth approximation of the compressive model. The reduction of effective gravitational force applying to a spherical body of radius \( R \) and density \( \rho \) is \( 3\rho h R/4 \) (Budding, 2005, Eqns 18, 26), in the first order of the Laplace screening parameter \( h \). With the mean density of Mercury as 5.427 and radius 2.440\( \times 10^8 \) (cgs), the value of \( h \) required to explain the discrepancy turns out to be a high \( 8 \times 10^{-12} \); that should have been measured by other means (cf. Edwards, 2002). The estimate of \( h \sim 10^{-15} \) speculated on by Budding (2005) appears another two or three orders of magnitude below what is currently measurable by radar delay measurements.

\(^1\)The quantity actually determined to have a 0.5% excess by Shapiro et al. (1972) is a parameter \( \lambda \) that linearly combines both the relativistic coefficients \( \beta \) and \( \gamma \), written as \( \beta' \) and \( \gamma' \), say, such that \( \beta' \) and \( \gamma' \) are both unity when \( \beta \) and \( \gamma \) have their regular GTR meanings. The compressive theory of gravity referred to in the present article has no direct effect on \( \beta \), but it causes the observed value of \( \gamma \) to become slightly greater than the \( \gamma \) of GTR. This would increase \( \lambda \) by 2/3 of the same proportion.
Towards determination of the Laplace gravity parameter $h$

References


Comment on
TOWARDS DETERMINATION OF THE
LAPLACE GRAVITY PARAMETER $h$

M. Przanowski

The paper is devoted to a compressive model of inertia and gravity. The problem with this paper is that no mathematical model has been done at all. Instead, the authors describe this model in the vague way. Consequently, with such a theory you can prove anything you want. The journal Concepts of Physics, as I understand it, is open to new ideas in physics, but the considerations should respect the methodology of physics and mathematics. Therefore, any new model should have clear mathematical representation. (According to famous words by P.A.M. Dirac: "Physical theory should have mathematical beauty"). The model presented in the paper does not fulfill that criterion. Hence, I do not recommend it for publication.
Authors’ response

We have been considering how best to deal with the general relativity subsection of the previously submitted paper, given its relatively simple and direct main aims.

We have thus added an additional page to the text and five new equations (6-10). The new part summarizes the conventional tensor approach to Einstein’s law of gravitation, leading up – in Equation (10) – to the expected Schwarzschild form, that re-appears as Equation (17). We had justified this previously through Equations (13)-(16), simply by considering local incremental modifications to the metric from Special Relativity. What this approach did not do explicitly was to show that the resulting formulae are consistent with the field equations for a generalized four-dimensional continuum and covariant force transformation. I believe we have now established this point, by spelling out the relevant equations and citing appropriate references.

But another point, of special relevance to our model for gravitation, is that the potential coefficients that appear in Equation (17) and introduced in Equations (13) and (14) are identical to the formulae (11) and (12) in the zero’th order of the compression model. The very slight difference in (11) and (12) that will appear when gravitational screening is taken into account can be treated as a linear perturbation, certainly to the order of accuracy that is currently measurable, and probably for quite some time to come.

We thus hope that we have (a) interpreted the comments of your referee appropriately, and (b) dealt with them in a suitable way so as to allow that there is no contradiction between the GTR law of gravitation and a compressive model in the zero’th order, and a very slight difference in the first order. We therefore resubmit the article with this in mind. Of course, we are ready to consider any further comments that may arise.

Edwin Budding and Oktay Yilmaz
ON A CONSISTENT QUANTUM ADIABATIC THEORY OF MOLECULES

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Abstract

We point out certain inconsistency in the foundations of the standard adiabatic method in quantum theory of molecules. As an alternative, we develop a particular approach that overcomes the appointed inconsistency. Based on this new approach, some interesting physical results come to the scope. First, we point out that the adiabatic method is substantially state-of-the-molecule dependent. E.g., the method distinguishes the definite conformations as a kind of the preferred states of a molecule. Second, the existence of the local minima for the effective potential for the nuclei system appears as a kind of a necessary condition for the validity of the adiabatic method. However, our approach does not fully answer...
the fundamental problem of the origin and stability of the definite (semi-classically well-defined) conformations of the large molecules. To this end, a new approach/theory is needed—as recently proposed within the context of the decoherence theory.
1 Introduction

Quantum mechanics of molecules (or quantum chemistry) relies on the adiabatic approximation. However, the "adiabatic approximation" is not unique a method. Actually, the two not-quite-equivalent methods are in parallel use. First, the method based on the adiabatic theorem \[ \kappa = \frac{m_A E_B}{m_B E_A}, \]
where \( m \) denotes the masses and \( E \) the energies of the systems \( A \) and \( B \). If \( \kappa \) is much less/greater than unity, then the adiabatic approximation may work well. On the other side, the Born-Oppenheimer approximation [1-3] employs solely the mass-ratio \( m_A/m_B \) as a criterion for the validity of the approximation. In order to reconcile the two approaches, the Born-Oppenheimer method should be completed by the proper boundary conditions. E.g., for an atom, the mass ratio \( m_e/m_p \), where \( e \) stands for the electron and \( p \) for the proton, is of the order of the adiabatic parameter for the hydrogen molecule. Yet, the protons in the atomic (non-point-like) nucleus are much faster than the atomic electrons, and the adiabatic approximation fails. The point is to introduce the boundary conditions for the protons, and thus realize that the proper adiabatic criterion (parameter) is the one stemming from the adiabatic theorem.

On the other side, the application of the adiabatic approximation is not fully consistent regarding the different molecular degrees of freedom. Actually, while the adiabatic parameters are typically defined relative to the original electrons- and the atomic-nuclei- degrees of freedom (the respective position-observables \( \hat{r}_e \) and \( \hat{r}_N \)), the canonical transformations of the molecular degrees of freedom are employed without an independent analysis of the validity of the adiabatic approximation for the new degrees of freedom.

The existence of the definite molecular conformations (geometrical shapes) is generally used as a kind of a sufficient condition for the adiabatic approximation (cf. below) and is usually taken for granted. On the other side, existence of the local minima (that are usually interpreted as the stable conformations, in agreement with the general phenomenology of the large molecules, e.g. of biomolecules) is also taken for granted, without a theoretical derivation from the first principles (to this end cf. Refs. [4, 5]).

In this paper, we propose a slight change (at variance with the approach of Ref. [4]) in the standard procedure in quantum chemistry.
and, in effect, we obtain both (i) the existence of the conformations as the preferred states within the adiabatic-approximation-method as well as (ii) we derive the existence of the local minima without resorting to phenomenology whatsoever.

2 The standard procedure: an inconsistency

In quantum chemistry, a molecule is defined as a collection of the electrons and the set of the atomic nuclei \((E + N)\). Usually, then the adiabatic parameter \(\kappa = m_e/m_\nu - e\) standing for an electron and \(\nu\) for a (typical) nucleus invokes the validity of the adiabatic approximation (valid also due to the adiabatic-theorem criterion–cf. above).

Formally, a molecule is originally defined by the following Hamiltonian:

\[
\hat{H} = \sum_i \hat{T}_{ei} + \sum_\alpha \hat{T}_{N\alpha} + \hat{V}^{(ee)}_{\text{Coul}} + \hat{V}^{(NN)}_{\text{Coul}} + \hat{V}^{(eN)}_{\text{Coul}} \quad \text{where}
\]

\[
\hat{V}^{(ee)}_{\text{Coul}} = k \sum_{i,j} |\hat{r}_{ei} - \hat{r}_{ej}|^{-1}, \quad k = e^2/4\pi\epsilon_0
\]

\[
\hat{V}^{(NN)}_{\text{Coul}} = k \sum_{\alpha,\beta} Z_{\alpha} Z_{\beta} |\hat{r}_{N\alpha} - \hat{r}_{N\beta}|^{-1}
\]

\[
\hat{V}^{(eN)}_{\text{Coul}} = -k \sum_{i,\alpha} Z_{\alpha} |\hat{r}_{ei} - \hat{r}_{N\alpha}|^{-1}.
\]

In eq. (1), \(\hat{T}\) stands for the kinetic terms (the index \(e\) standing for electrons, and \(N\) for the nuclei) while \(\hat{V}\) standing for the Coulomb interactions, where appear the original position-observables, \(\hat{r}_{ei}, \hat{r}_{N\alpha}\) of the \(i\)th electron and of the \(\alpha\)th atomic nucleus, respectively.

The semiclassical estimates justify the following general procedure: (a) from the electrons point of view, the nuclei-system is rather slow, and virtually immovable for some period of time, thus allowing one (in the zeroth order) to "freeze" the nuclei dynamics, and (b) from the nuclei-system point of view, the electrons are too fast and the nuclei dynamics may be defined by an effective Hamiltonian stemming from the averaging of the molecule’s Hamiltonian over the
electrons state. Existence of the definite electrons state stems from the exact state for the $E + N$ system [1-3]:

$$|\Psi\rangle_{e+N} = |\phi_n(Q)\rangle_e |\chi \rangle_N + |O(\kappa^{3/4})\rangle_{e+N}. \tag{2}$$

Needless to say, the fixed values of the nuclei positions give rise to the transcription: $\hat{r}_{N\alpha} \rightarrow \hat{r}_{N\alpha} \hat{I}$ (where $\hat{I}$ stands for the identity operator). I.e., instead of the observables, we obtain their fixed eigenvalues (appearing now as the parameters for the electrons system) for every index $\alpha$. Certainly, the fixed values $\hat{r}_{N\alpha}$ give directly rise to a definite position of the center-of-mass of the nuclei system, as well as to the definite geometrical shape (conformation, $Q$) of a molecule. So, in a sense, the existence of the definite (unique) conformation of a molecule represents a kind of a sufficient condition for the adiabatic approximation, which is formally defined by eq. (2) and by the following expressions.

Actually, for the fixed spatial positions of the nuclei, one may neglect the nuclei-dynamics, thus redefining the Hamiltonian eq. (1) to obtain the electrons-system Hamiltonian:

$$\hat{H}_e = \hat{T}_e + \hat{V}_{ee} + \hat{V}_{N} + \text{const} \hat{I}, \tag{3}$$

where appear the electrons-system observable following from the above transcription $\hat{V}_e^N = -k \sum_{i,\alpha} Z_\alpha |\hat{r}_{ei} - \hat{r}_{n\alpha} \hat{I}|^{-1}$, and the constant term $\text{const} \equiv k \sum_{\alpha,\beta} Z_\alpha Z_\beta |\hat{r}_{N\alpha} - \hat{r}_{N\beta}|^{-1}$; $\hat{I}$ is the identity operator. Then one obtains the Schrodinger equation for the (time independent) Hamiltonian eq. (3), where the eigenstates can be denoted as $|\phi_n(Q)\rangle_e$ and the corresponding eigenvalues as $E^e_n(Q)$, for the different, fixed values of the nuclei positions collectively denoted as $Q$.

Solving the Schrodinger equation for $\hat{H}_e$ gives rise to the $Q$-parameterized electrons-energy eigenvalues $E^e_n(Q)$, and for the frozen nuclei the exact state eq. (2) reads:

$$|\Psi\rangle_{e+N} = |\phi_n(Q)\rangle_e \otimes_{\alpha} |\hat{r}_\alpha \rangle_N, \quad \hat{r}_{N\alpha} |\hat{r}_\alpha \rangle_N = \hat{r}_{N\alpha} |\hat{r}_\alpha \rangle_N. \tag{4}$$

Now, averaging over the electrons state (that is a solution of the Schrodinger equation for the Hamiltonian eq. (3)), gives rise to the effective Hamiltonian for the nuclei system as defined by:
\[ \hat{H}_N^{\text{eff}} = e \langle \hat{H} \rangle_e = E_n^e(Q) + \hat{T}_N + W\hat{I}, \quad W \ll 1. \quad (5) \]

The inconsistency implicit in eq. (5) is as follows. In order to obtain \( E_n^e \) in eq. (5), one should: (i) apply the transcription (cf. above) \( \vec{r}_{N\alpha} \rightarrow \vec{r}_{N\alpha}\hat{I} \), i.e. to ”freeze” the nuclei system, and still (ii) not to neglect the nuclei kinetic energy, \( \hat{T}_N \)–in contradistinction with the derivation of eq. (3). While (i) and (ii) are implicit to derivation of eq. (5), it is really strange to investigate the dynamics of a system by freezing the system’s position (the above point (i)), and still to employ the kinetic term for the ”frozen” system (the above point (ii)).

This inconsistency is the true objective of the present paper. Bearing in mind that the adiabatic approximation works well for the plenty of the physical models and physical situations, we wonder if its physical contents (formally presented by the expressions eqs. (2)–(5)) can be obtained while avoiding the above-distinguished inconsistency. Finally, we emphasize another weak point of the standard adiabatic approximation (recently criticized in Ref. [4]): the existence of the local minima in the effective potential \( E_n^e(Q) \) for the nuclei system does not stem from the first principles–rather, the minima are stipulated in order to fit the theory with the general experimental data.

As we show below, there exists a simple procedure that avoids the inconsistency, while giving rise to both, re-deriving the adiabatic approximation eqs. (2)-(5), as well as providing us with the interested theoretical rewards, such as the quantum-mechanical ”derivation” of the local minima for \( E_n^e(Q) \).

Physically, the result of the adiabatic approximation is truly remarkable. First, by reducing the exact Hamiltonian eq. (1) to the sum \( \hat{H}_e + \hat{T}_N \) (as implicit to eq. (5)), it makes the system \( E + N \) as a pair of noninteracting systems; the exact interaction \( \hat{V}_{eN} \) in eq. (1) reduces to the external field for the electrons system (cf. \( \hat{V}_{e}^N \) in eq. (3)), and thus produces the effective external potential for the nuclei system \( E_n^e(Q) \) in eq. (5). Second, the interaction \( \hat{V}_{eN} \) that in general produces quantum entanglement (i.e. nonseparability of the subsystems and nonlocality) is managed to be written effectively as a sum of the two external potentials for the subsystems, which therefore appear effectively as approximately noninteracting systems. Needless
to say, then the total energy of a molecule is (approximately) a sum of the energies of the two systems.

However, as it directly follows from our approach, this is a state-dependent result—a similar argument has recently been raised in Ref. [4]. So, the domain of applicability of the adiabatic approximation is limited: in general, the subsystems $E$ and $N$ are mutually entangled, and the expression eq. (2) refers to the very special states of the molecule.

3 Removing the inconsistency

Our task reads: starting from the model eq. (1), to develop a procedure that, while avoiding the above-distinguished inconsistency, can still lead to the expressions of the standard adiabatic theory, eqs. (2)-(5).

Therefore, we start from the standard wisdom of the adiabatic theorem that is expressed by the points (a) and (b) of Section 2, and formally presented by the Hamiltonian eq. (3) and its corresponding Schrodinger equation:

$$\hat{H}_e|\phi_n(Q)\rangle = E^e_n(Q)|\phi_n(Q)\rangle_e,$$  
and the corresponding state eq. (4) for the composite system $E + N$.

So, our starting point is the derivation of eq. (5).

To this end, we propose the following procedure: we add and subtract the terms $\hat{V}_N^e$ and $const\hat{I}$ (that appear in eqs. (3) and (5)) to the total Hamiltonian $\hat{H}$ eq. (1). The expression obtained to be substituted in $\langle \hat{H} \rangle_e$ eq. (5). Then the rhs of eq. (5) reads:

$$\langle \hat{H} \rangle_e = \langle \hat{H} + \hat{V}_N^e + const\hat{I} - \hat{V}_N^e - const\hat{I} \rangle_e.$$  

Formally, our procedure bears some similarity with the related proposal in Ref. [4]. After a closer inspection, it is clear that the two procedures do not have much in common, mathematically and/or physically.

Bearing in mind eq. (3), the rhs of eq. (7) obtains the following form:

$$\hat{H}_N^{eff} = E^e_n(Q) + \hat{T}_N + W\hat{I} + \langle \hat{V}_e^N - \hat{V}_e^N + \hat{V}_N^N - const\hat{I} \rangle_e.$$  

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The effective nuclei-system Hamiltonian eq. (8) differs from the original one eq. (5) by the presence of the last term on the rhs of eq. (8). Therefore, in order to deduce the standard expression eq. (5), the last term in eq. (8) should equal (at least approximately) to zero. More precisely: the desired equality to zero can not follow in the operator form—there appear the difference of the operators and of the constant terms (the c-numbers)—and therefore one should express this condition as the two simultaneously fulfilled equalities for a given state $|\chi\rangle_N$ for the nuclei system:

$$e\langle \hat{V}^{eN} - \hat{V}^{eN}_e | \chi\rangle_N = 0,$$

and

$$e\langle \hat{V}^{NN} - \text{const}\hat{I} | \chi\rangle_N = 0.$$

(10)

Apparently, only the special set of states $|\chi_i\rangle_N$ can fulfill the equalities eqs. (9) and (10)—the adiabatic method is state dependent. As the obvious candidate in this regard appears the state eq. (4), i.e. the state of the "frozen" nuclei system:

$$|\chi\rangle_N = \otimes_\alpha |\vec{r}_{N\alpha}\rangle_N.$$

(11)

Certainly, the state eq. (11) is not an eigenstate of either of the effective Hamiltonians eq. (5) and/or eq. (8).

So, we obtain the expression eq. (11)—i.e. the existence of the definite molecules conformation(s) as implicit in the state eq. (11) and in the collective variable $Q$—as a kind of a necessary condition for the validity of eq. (5)—eq. (8) implies eqs. (9) and (10), which, in turn, imply eq. (11). Therefore, one may say, that the rhs of eq. (11) appears in the standard approach (cf. also eq. (4)) as a kind of a sufficient condition, while within our approach it appears as a kind of a necessary condition for the validity of eq. (5).

In the next section we emphasize that the choice of the state eq. (11) is unique as long as one deals with the exact equality in eqs. (9) and (10). For the approximate equalities in eqs. (9) and (10), one can obtain the physically convenient normalizable states instead of the state eq. (11), and to investigate the condition of the validity of eqs. (9) and (10) for virtually arbitrary instant of time $t$. Bearing this in mind, it becomes clear that the kinematic conditions eqs. (9)
and (10) do not introduce the frozen nuclei eq. (11)–the situation we wanted to avoid by the very conditions eqs. (9) and (10). The full meaning of this condition comes only from the dynamical analysis performed in the next section.

3.1 The dynamical conditions

The conditions eqs. (9) and (10) refer to a fixed instant of time. So, in order to be able within our approach to deal with eq. (5) for "every" instant of time, the states $|\chi(t)\rangle_N$ should keep the equalities eqs. (9) and (10) valid virtually for every instant of time $t$.

At the first sight, this seems to be a mission impossible. Actually, one should provide the states $|\chi(t)\rangle_N$ that are dynamically produced by the evolution generated by both $\hat{H}^{eff}_N$ and $\hat{H}^{teff}_N$, and to fulfill both the (at least approximate) equalities in eqs. (9) and (10). Interestingly and surprisingly enough, there exists such a model that naturally appears within our approach. To this end, we directly employ some general quantum mechanical results without resorting to any phenomenological criteria and/or to any additional (re)interpretation of the obtained results.

The state on the rhs of eq. (11) represents the quantum-mechanical counterpart of the exact positions of the nuclei in the classical configuration space of the nuclei system. As it is well known, these un-normalizable states produce the problems in a quantum-mechanical description of the "bound" physical systems, for which the normalizable states are needed. To this end, the correspondence of the unnormalizable and the normalizable states has been developed by von Neumann [6] in his theory of "macroscopic" measurements (cf. Appendix A). In effect, the unnormalizable state $|\vec{r}\rangle$ is exchanged by a gaussian (the minimal-uncertainty, i.e. the wave-packet, i.e. the "coherent") state $|\psi_{qp}\rangle$, where $q$ is the mean value of the position- and $p$ is the mean value of the momentum- operator for the system. A similar method has been extensively used by Omnes [7] within the microlocal analysis in the semiclassical approach to the quantum-mechanical description of the many-particle systems.

As it is distinguished in Appendix A, one may apply the following exchange of states:

$$\otimes|\vec{r}_\alpha\rangle_N \rightarrow |\Psi_{qp}\rangle_N$$

(12)
where appear the mean values $q$ and $p$ of the collective position observable $\hat{Q}$ and of its conjugate (collective momentum) observable $\hat{P}$ for the nuclei system, respectively. More precisely, instead of eq. (11), one may write

$$|\chi\rangle_N = |\Psi_{qp}\rangle_N,$$

while bearing in mind the only approximate (yet satisfactory—cf. Appendix A) equalities in eqs. (9) and (10).

Needless to say, up to the uncertainty relations for the collective observables $\hat{Q}$ and $\hat{P}$, a normalizable state $|\Psi_{qp}\rangle_N$ bears the definite (semitically well-defined) molecular conformation as defined by the mean value $q$ of the collective observable $\hat{Q}$. This way we fulfill the approximate equality of the effective Hamiltonians for the nuclei system for the given instant of time, $t = 0$. Let us denote the state for this instant of time as $|\Psi_{q_0,p_0}\rangle_N$. So, there remains the task of maintaining the validity of the approximate validity of eqs. (9) and (10) for the time instants $t > 0$.

Interestingly, and somewhat surprisingly enough, there exist the conditions that can fulfill the requirement of the virtual non-change of a gaussian $|\Psi_{qp}\rangle_N$ in the course of the both dynamics generated by the effective Hamiltonians eqs. (5) and (8).

Actually, as Hagedorn [8] (cf. also Omnes [7], p. 266, for the simpler mathematical formulation) shows: the Schrödinger equation preserves the initial gaussian state if the external potential is a harmonic potential (cf. Ref. [9] for the similar result regarding the central—e.g. the Coulomb, or the Kepler–field). More precisely: governed by the Schrödinger equation for the harmonic potential, the initial gaussian state $|\Psi_{q_0,p_0}\rangle_N$ evolves in time as:

$$|\Psi_{q_0,p_0}\rangle_N \rightarrow |\Psi_{q(t),p(t)}\rangle_N,$$

so that the classical dynamics $(q_0, p_0) \rightarrow (q(t), p(t))$ is governed itself by the harmonic potential—cf. Appendix B.

In the context of our considerations: the validity of eq. (14) effectively implies that the potential $E_n^c(Q)$ should be a polynomial of the second order in $Q$. Bearing in mind that the gaussians provide the eqs. (9) and (10) approximately to be valid, we may emphasize: if the
nuclei-system initial state is a gaussian, then the two effective Hamiltonians $\hat{H}_N^{\text{eff}}$ and $\hat{H}_N^{\text{eff}}$ mutually coincide, and the nuclei-system state is a gaussian in every instant of time—the gaussian states give rise to eq. (5) as desired.

The mean value $q$ refers to the equilibrium positions of the nuclei system. So, one may conclude that the nuclei system evolves in time as the continuous change of the conformation of the molecule, where the change of the conformation (i.e. of the mean value $q$) follows the classical harmonic oscillations. This gives an appealing physical picture to be discussed below.

Finally, as another benefit appears a truly interesting result. The second-order polynomials bear the extremal point, i.e. the minimal value we are only interested in. Formally, such a polynomial represents a ”potential well” with the well-defined bottom of the well. Within the standard approach, existence of the local minima in the potential $E_n^c(Q)$ is a phenomenological rule. However, within our approach, the existence of a minimum stems from the first principles: the validity of eq. (5) implies the approximate validity of eqs. (9) and (10), whose dynamical validity (referring to the nuclei-system) effectively—the validity of eq. (14) is known only for the harmonic potential, as well as (non-interesting for us) for the central-field potential—requires the harmonic potential for $E_n^c(Q)$, which unavoidably bears the well-defined bottom (the minimum value).

3.2 The origin and the stability of the molecular conformations: a problem

Within our approach, the adiabatic approximation prefers the definite conformational states of the molecule’s nuclei system, in contradistinction e.g. with the atomic physics, in which the definite energy-states are preferred. E.g., the initial wave packet centered in the vicinity of the bottom of a potential well satisfies both, the equality of $\hat{H}_N^{\text{eff}}$ eq. (5) and $\hat{H}_N^{\text{eff}}$ eq. (8), as well as bears stability of its form in the course of time, eq. (14). This time interval equals infinity for the exactly harmonic potential for the molecular conformations $Q$ [7, 8].

In the vicinity of the bottom of the potential well, the mean value $q = \langle \hat{Q} \rangle$ (that defines the molecule conformation) changes in accordance with the classical law for the harmonic oscillator. Physically, it
means that, in the vicinity of the local minimum (for which, as we see, the adiabatic method is expected to work properly), one does not deal with the unique conformation. Rather, there appears the continuous, classical-physics-described harmonic change of the molecule conformation. Certainly, for the small amplitudes, the different conformations can not be macroscopically distinguished, while they bear the slightly different mean-energies, \( N \langle \Psi_{q(t)p(t)} | \hat{H}_{eff}^N | \Psi_{q(t)p(t)} \rangle_N = f(t) \).

In general (cf. below), the coherent superpositions of the wave packets need not satisfy the conditions eq. (9) and (10). So, we conclude that the adiabatic approximation need not be valid for the states out of the local minimum and/or for the arbitrary superpositions of the gaussian (wave packet) states.

For the realistic systems, one can hardly expect the exact harmonic character of the potential—typically, there appear the non-negligible anharmonic terms in the potential. In this case, needless to say, the gaussian states do not satisfy ”robustness” as defined by eq. (14). In effect, the dynamics of the initial gaussian is transformed into a coherent superposition of the different gaussian states. On the other side, our considerations do not forbid the existence of more-than-one local minimum for the conformation-system potential. While our analysis establishes validity of the general results, eqs. (2)-(5), for every single local minimum (the bottom of a potential well), it does not fully describe the dynamics of the arbitrary superpositions of the gaussian states—cf. the next section.

The following question is fundamental in quantum mechanics of molecules [10]: why certain molecules (e.g. macromolecules) usually bear the definite conformation, instead of having the definite energy? To this end, our approach offers the following yet incomplete answer: if the initial state is a gaussian (centered around the very bottom of the potential well), then in the exact harmonic potential it will remain a gaussian ”forever”—to this end, the gaussian states bear robustness (the dynamical stability) as expected for the large-molecules conformations. However, as we show below, even certain coherent superpositions of the gaussian states can satisfy eq. (5) as well formally as the ”robustness”, thus not allowing one to ascribe a definite conformation to a molecule in such states. In general, however, the arbitrary coherent superpositions of the gaussian states—as it can be expected—do not satisfy eq. (5), thus giving rise to both, the
limitation of the adiabatic method within our approach, as well as leaving the above-distinguished problem open. To this end, it seems that a new theory is needed—it seems that the so-called decoherence theory might clear the terrain in a consistent way [11].

4 Discussion

We try to avoid the inconsistency implicit in the derivation of eq. (5). To this end, we employ certain general quantum-mechanical results—cf. Appendices A and B. This gives rise (cf. eqs. (9) and (10)) to the quantum-state-dependent applicability of the general formulas eq. (2)-(5) of the standard adiabatic method in quantum mechanics of molecules. In effect, the existence of a minimum of the effective potential for the nuclei system appears as a kind of the necessary condition for the method—apart from the non-interesting central field, the harmonic potential appears as the only candidate for the validity of eq. (14). Interestingly enough—and in contradistinction with the standard approach—this result follows from the first principles, without resorting to phenomenology.

Our approach does not per se forbid the coherent superpositions of the different conformations states, neither it establishes the unique local minimum for the potential. Interestingly enough, certain superpositions of conformations can still match the adiabatic method—i.e. to satisfy both, eqs. (9) and (10), as well as eq. (14)—cf. Appendix C. [Some of these superpositions might be recognized as the energy eigenstates for the vibrational degrees of freedom of the molecule’s nuclei-system (for comparison cf. the model of the small-molecules chirality [12, 13]).]

However, the arbitrary superpositions of the conformational states do not justify eq. (5), and this is the instance at which our considerations departure from the standard adiabatic method—then the effective Hamiltonians eq. (5) and eq. (8) are not mutually equal. So, in such cases, one is obliged to deal with the exact Hamiltonian eq. (1), which, while bearing the interaction term $V^{eN}$, gives in general rise to the quantum entanglement in the composite system $E + N$ [12]. Having this in mind, it becomes evident how restrictive, and how-much-convenient is the state eq. (2).

Therefore, the following (mutually related) questions remain open within the standard, likewise within our approach: why, after all, the
initial state of the nuclei system would be a gaussian? (the universally valid quantum mechanics does not set any limitations in this regard—cf. also the Ref. [10]); how can we treat the arbitrary superpositions of the gaussian states as the possible cases not properly described by the adiabatic approximation?; why the small molecules are usually described by energy (not yet bearing the definite conformation), and the large molecules are typically described by their spatial conformations (without the definite energy)—the former being qualitatively modeled by the state eq. (21) of Appendix C, while the later being described by eq. (13)? To this end, we believe, a new physical theory is needed as recently proposed and elaborated in Ref. [11]—that is our conclusion.

An independent analysis concerning the alternative description of a molecule by introducing the nuclei-system center-of-mass and the "relative coordinates" (that directly introduce the molecule conformation) will be separately presented.

**Appendix A**

For simplicity, let us consider a one-dimensional system defined by the continuous position-observable $\hat{x}$ and its conjugate (also continuous) momentum $\hat{p}$; $[\hat{x}, \hat{p}] = i\hbar$.

"Coarse graining" of the real axis into the segments of the width $\delta x$ can give the approximations of the exact position-observable eigenstates $|x\rangle$. Actually, every segment of the real axis $[q-\delta x/2, q+\delta x/2]$, centered around the value $q$, can be properly approximated by a gaussian state $|\Psi_{qp}\rangle$, if $\langle \hat{x} \rangle = q$ and $\langle \hat{p} \rangle = p$, while the standard deviation of $\hat{x}$ in this state, $\Delta \hat{x}$, is of the order of $\delta x$. This way, the exact eigenvalues $x$ of $\hat{x}$ and the corresponding eigenstates $|x\rangle$ are properly approximated by $q$ and $|\Psi_{qp}\rangle$, respectively. An extensive analysis in this regard can be found in Omnes [7].

The form of the gaussian states does not depend on the system’s dimensionality. The set of these states is overcomplete and there are not the exactly-orthogonal pairs of these states. Nevertheless, even if orthonormalized (then defining a nonunique orthonormalized basis $|\Psi_{\mu\nu}\rangle$), these states maintain the approximate quantum-mechanical description of the exact position-observable eigenstates.
Actually, von Neumann [6] shows that the observables $\hat{x}$ and $\hat{p}$ can be properly approximated by the discrete observables $\hat{\xi}$ and $\hat{\pi}$, $[\hat{\xi}, \hat{\pi}] = 0$, and their common eigenstates $|\Psi_{\mu\nu}\rangle$ constitute an orthonormalized basis. Interestingly enough, the minimal uncertainty condition $\epsilon\eta = \frac{\hbar}{2}$ is only linearly increased. Namely, one can write:

$$\|(\hat{x} - \xi)|\Psi_{\mu\nu}\rangle\| \leq C\epsilon, \quad \text{and}$$  \hspace{1cm} (15)

$$\|(\hat{p} - \pi)|\Psi_{\mu\nu}\rangle\| \leq C\eta,$$  \hspace{1cm} (16)

while $C < 60$ [6].

So, even the more general states $|\Psi_{\mu\nu}\rangle$ can be used in order (nonuniquely) to approximate the position-observable eigenstates. For the reasons made explicit in the body text, we shall further refer to the non-orthogonal gaussian states $|\Psi_{qp}\rangle$ for the purpose of approximating the position-observable exact eigenstates $|\vec{r}\rangle$, as assuming:

$$\|e\langle \hat{V} e^N \rangle |\Psi_{qp}\rangle_N \| \ll 1, \quad \|e\langle \hat{V} NN - \text{const.}\hat{I} \rangle |\Psi_{qp}\rangle_N \| \ll 1.$$  \hspace{1cm} (17)

**Appendix B**

For a system of the arbitrary number of the degrees of freedom, denoted collectively as $\hat{Q}$, Hagedorn [8] (see also Omnes [7], p. 266) was able to show that the Schrödinger dynamics preserves the gaussian states as:

$$\exp(-it\hat{H}/\hbar)|\Psi_{q_0p_0}\rangle = |\Psi_{q(t)p(t)}\rangle,$$  \hspace{1cm} (18)

if for the Hamiltonian $\hat{H} = \hat{T} + U(\hat{Q})$–the potential $U(\hat{Q})$ is at most the second-order polynomial in $Q$, while the classical dynamics $(q_0, p_0) \to (q(t), p(t))$ is governed by the classical counterpart of the harmonic potential $\hat{U}$; $q = \langle \hat{Q} \rangle$ and $p = \langle \hat{P} \rangle$, while $[\hat{Q}, \hat{P}] = \imath\hbar$. Physically, the mean values of both $\hat{Q}$ and $\hat{P}$ follow the harmonic-oscillator trajectories in the classical phase space, while the quantum oscillations (the observable $\hat{Q}$) refer to instantaneous conformation (i.e. the equilibrium position) $q$ in an instant of time $t$. 
The second order-polynomial \( U(Q) \) defines a "potential" well with the well defined minimum—i.e. the bottom of the well. Now, according to the Legend-Dirichle theorem, the very bottom of the potential represents a stable point (the equilibrium point) for the harmonic oscillations around the bottom. Physically, the bottom of the potential well formed by the parabolic potential \( U(Q) \) represents the one (not necessarily the unique) stable conformation of the molecule. Needless to say, this way we conclude that existence of the local minima in the effective potential \( U(Q) \) for the molecule nuclei-system stems from the first principles, without calling for any phenomenology whatsoever.

Appendix C

Let us rewrite eqs. (9) and (10) as:

\[
e\langle \hat{V}^eN - \hat{V}^e_e \rangle e|\Psi_{qp}\rangle_N = |\epsilon\rangle_N, \quad \epsilon \equiv \|\epsilon\rangle_N\| \ll 1 \quad (19)
\]

and

\[
e\langle \hat{V}^N - \text{const}\hat{I} \rangle e|\Psi_{qp}\rangle_N = |\epsilon'\rangle_N, \quad \epsilon' \equiv \|\epsilon'\rangle_N\| \ll 1. \quad (20)
\]

Let us consider a superposition

\[
|\chi\rangle_N = c_1|\Psi_{q_1p_1}\rangle_N + c_2|\Psi_{q_2p_2}\rangle_N, \quad (21)
\]

where \(|\Psi_{qp}\rangle_N\)s satisfy eqs. (19) and (20); the corresponding \(\epsilon\)s can be denoted respectively as \(\epsilon_1\) and \(\epsilon_2\). Then one can easily prove the inequality:

\[
\|e\langle \hat{V}^eN - \hat{V}^e_e \rangle e|\chi\rangle_N\| \leq 2\epsilon \ll 1, \quad (22)
\]

where \(\epsilon\) is the average for \(\epsilon_i, i = 1,2\). On the other side, the state \(|\chi\rangle_N\) satisfies the Schrodinger law, as a consequence of the linearity of the Schrodinger law, and in accordance with eq. (14) for \(|\Psi_{qp}\rangle\)s appearing in eq. (21). So, one may conclude that even certain coherent superpositions of the conformational states can satisfy eq. (5), i.e. justify applicability of the adiabatic approximation.
However, the arbitrary coherent superpositions,
\[ \sum_{i=1}^{L} c_i |\Psi_{q_i,p_i}\rangle, \]  
(23)
need not satisfy eq. (5) as—in analogy with eq. (22)—one obtains \( L\epsilon \) that need not be a small number. Since all the analogous conclusions can follow for eq. (20), one can conclude that the arbitrary superpositions of the gaussian states need not satisfy eq. (5)—i.e., as it stems from eqs. (8)-(10), the adiabatic approximation need not be applicable for the arbitrary superpositions of the gaussian states.

References


GRAVITATIONAL INTERACTION OF QUANTUM LEVEL AND CONSEQUENCES THEREOF

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Abstract

Parity no conservation in the $\beta$-decay processes is considered as fundamental property of weak interactions. Nevertheless, this property can be treated as anomaly, because in fundamental interactions of the rest types parity is conserved. Analogously, anomaly in the short-duration strong-current pulse discharges is well known. The essence of this phenomenon consists in generation of local high-temperature plasma formations with the typical values of its thermodynamical parameters exceeding those related to the central section of a discharge. In this paper, an attempt is undertaken to treat these anomalies as manifestation of fundamental properties of gravitational emission. Some consequences of this assumption can be tested in the $\beta$-decay experiments as well as in the experiments with short-duration $z$-pinch-type pulse discharges.
The notion of gravitational emission as an emission of the same level with electromagnetic emission is based on the proven fact of existence of electron’s stationary states in its own gravitational field, characterized by gravitational constant $K=10^{42}G$ ($G$ – Newton’s gravitational constant).
1 Gravitational emission of electrons with a banded spectrum as emission of the same level with electromagnetic emission.

For a mathematical model of interest, which describes a banded spectrum of stationary states of electrons in the proper gravitational field, two aspects are of importance. First, in Einstein’s field equations $\kappa$ is a constant which relates the space-time geometrical properties with the distribution of physical matter, so that the origin of the equations is not connected with the numerical limitation of the $\kappa$ value. Only the requirement of conformity with the Newtonian Classical Theory of Gravity leads to the small value $\kappa = 8\pi G/c^4$, where $G, c$ are, respectively, the Newtonian gravitational constant and the velocity of light. Such requirement follows from the primary concept of the Einstein General Theory of Relativity (GR) as a relativistic generalization of the Newtonian Theory of Gravity. Second, the most general form of relativistic gravitation equations are equations with the $\Lambda$ term. The limiting transition to weak fields leads to the equation

$$\Delta \Phi = -4\pi \rho G + \Lambda c^2,$$

where $\Phi$ is the field scalar potential, $\rho$ is the source density. This circumstance, eventually, is crucial for neglecting the $\Lambda$ term, because only in this case the GR is a generalization of the Classical Theory of Gravity. Therefore, the numerical values of $\kappa = 8\pi G/c^4$ and $\Lambda = 0$ in the GR equations are not associated with the origin of the equations, but follow only from the conformity of the GR with the classical theory.

From the 70’s onwards, it became obvious [1] that in the quantum region the numerical value of $G$ is not compatible with the principles of quantum mechanics. In a number of papers [1] (including also [2]) it was shown that in the quantum region the coupling constant $K$ ($K \approx 10^{40}, G$) is acceptable. The essence of the problem of the generalization of relativistic equations on the quantum level was thus outlined: such generalization must match the numerical values of the gravity constants in the quantum and classical regions.

In the development of these results, as a micro-level approximation of Einstein’s field equations, a model is proposed, based on the following assumption:
"The gravitational field within the region of localization of an elementary particle having a mass $m_0$ is characterized by the values of the gravity constant $K$ and of the constant $\Lambda$ that lead to the stationary states of the particle in its proper gravitational field, and the particle stationary states as such are the sources of the gravitational field with the Newtonian gravity constant $G$.

The most general approach in the Gravity Theory is the one which takes twisting into account and treats the gravitational field as a gage field, acting on equal terms with other fundamental fields ([3]). Such approach lacks in apriority gives no restrictions on the microscopic level. For an elementary spinor source with a mass $m_0$, the set of equations describing its states in the proper gravitational field in accordance with the adopted assumption will have the form

$$\left\{i\gamma^\mu(\nabla_\mu + \kappa \overline{\Psi} \gamma_\mu \gamma_5 \Psi)\gamma_5) - m_0 c/\hbar\right\} \Psi = 0, \quad (1)$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa \{T_{\mu\nu}(E_n) - \mu g_{\mu\nu} + (g_{\mu\nu}S_\alpha S^\alpha - S_\mu S_\nu)\}, \quad (2)$$

$$R(K, \Lambda, E_n, r_n) = R(G, E_n', r_n), \quad (3)$$

$$\left\{i\gamma^\mu \nabla_\mu - m_n c/\hbar\right\} \Psi' = 0, \quad (4)$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa' T_{\mu\nu}'(E_n') \quad (5)$$

The following notations are used throughout the text of the paper: $\kappa = 8\pi K/c^4$, $\kappa' = 8\pi G/c^4$, $E_n$ is the energy of stationary states in the proper gravitational field with the constant $K$, $\Lambda = \kappa \mu$, $r_n$ is the value of the coordinate $r$ satisfying the equilibrium n-state in the proper gravitational field, $\overline{\kappa} = \kappa_0 \kappa$, $\kappa_0$ is the dimensionality constant, $S_\alpha = \overline{\Psi} \gamma_a \gamma_5 \Psi$, $\nabla_\mu$ is the spinor-coupling covariant derivative independent of twisting, $E_n'$ is the energy state of the particle having a mass $m_n$ (either free of field or in the external field), described by the wave function $\psi'$ in the proper gravitational field with the constant $G$. The rest of the notations are generally adopted in the gravitation theory.

Equations (1) through (5) describe the equilibrium states of particles (stationary states) in the proper gravitational field and define the localization region of the field characterized by the constant that satisfies the equilibrium state. These stationary states are sources of the field with the constant $G$, and condition (3) provides matching the..."
solution with the gravitational constants and $G$. The proposed model in the physical aspect is compatible with the principles of quantum mechanics principles, and the gravitational field with the constants and $\Lambda$ at a certain, quite definite distance specified by the equilibrium state transforms into the filed having the constant $G$ and satisfying, in the weak field limit, the Poisson equation.

The set of equations (1) through (5), first of all, is of interest for the problem of stationary states, i.e., the problem of energy spectrum calculations for an elementary source in the own gravitational field. In this sense it is reasonable to use an analogy with electrodynamics, in particular, with the problem of electron stationary states in the Coulomb field. Transition from the Schrödinger equation to the Klein-Gordon relativistic equations allows taking into account the fine structure of the electron energy spectrum in the Coulomb field, whereas transition to the Dirac equation allows taking into account the relativistic fine structure and the energy level splitting associated with spin-orbital interaction. Using this analogy and the form of equation (1), one can conclude that solution of this equation without the term $\kappa \Psi \gamma_\mu \gamma_5 \Psi \gamma_5$ gives a spectrum similar to that of the fine structure (similar in the sense of relativism and removal of the principal quantum number degeneracy). Taking the term $\kappa \Psi \gamma_\mu \gamma_5 \Psi \gamma_5$ into account, as is noted in [1], is similar to taking into account of the term $\Omega^{\mu \nu} \Psi F_{\mu \nu}$ in the Pauli equation. The latter implies that the solution of the problem of stationary states with twisting taken into account will give a total energy-state spectrum with both the relativistic fine structure and energy state splitting caused by spin-twist interaction taken into account. This fact, being in complete accord with the requirements of the Gauge Theory of Gravity, allows us to believe that the above-stated assumptions concerning the properties of the gravitational field in the quantum region are relevant, in the general case, just to the gravitational field with twists.

Complexity of solving this problem compels us to employ a simpler approximation, namely,: energy spectrum calculations in a relativistic fine-structure approximation. In this approximation the problem of the stationary states of an elementary source in the proper gravitational field well be reduced to solving the following equations:

$$f'' + \left( \frac{\nu' - \lambda'}{2} + \frac{2}{r} \right) f' + e^\lambda \left( K^2 e^{-\nu} - K^2_0 - \frac{l(l+1)}{r^2} \right) f = 0, \quad (6)$$
\[-e^{-\lambda} \left( \frac{1}{r^2} - \frac{\lambda'}{r} \right) + \frac{1}{r^2} + \Lambda = \beta(2l + 1) \times \]
\[\times \left\{ f^2 \left[ e^{-\lambda} K_n^2 + K_0^2 + \frac{l(l + 1)}{r^2} \right] + f' r^2 e^{-\lambda} \right\} \] (7)
\[-e^\lambda \left( \frac{1}{r^2} + \frac{\nu'}{r} \right) + \frac{1}{r^2} + \Lambda = \beta(2l + 1) \times \]
\[\times \left\{ f^2 \left[ K_0^2 - K_n^2 e^{-\nu} + \frac{l(l + 1)}{r^2} \right] - e^\lambda f'^2 \right\}, \] (8)
\[\left\{ -\frac{1}{2} (\nu'' + \nu'^2) - (\lambda' + \nu') \left( \frac{\nu'}{4} + \frac{1}{r} \right) + \frac{1}{r^2} (1 + e^2) \right\} |_{r = r_n} = 0, \] (9)
\[f(0) = \text{const} \ll \infty, \] (10)
\[f(r_n) = 0, \] (11)
\[\lambda(0) = \nu(0), \] (12)
\[\int_0^{r_n} f^2 r^2 dr = 1. \] (13)

Equations (6)-(8) follow from equations (14)-(15)
\[\left\{ -g^{\mu\nu} \frac{\partial}{\partial x_\mu} \frac{\partial}{\partial x_\nu} + g^{\mu\rho} \Gamma^\alpha_{\rho\nu} \frac{\partial}{\partial x_\alpha} - K_0^2 \right\} \Psi = 0, \] (14)
\[R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa (T_{\mu\nu} - \mu g_{\mu\nu}) \] (15)

after the substitution of \(\Psi\) in the form: \(\Psi = f_{El}(r) Y_{lm}(\theta, \phi) \text{exp} \left( \frac{-iEt}{\hbar} \right)\) into them and specific computations in the central-symmetry field metric with the interval defined by the expression [4]
\[ds^2 = c^2 e^\nu dt^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) - e^\lambda dr^2. \] (16)

The following notation is used above: \(f_m\) is the radial wave function that describes the states with a definite energy \(E\) and the orbital momentum \(l\) (hereafter the subscripts \(El\) are omitted), \(Y_{lm}(\theta, \phi)\) are spherical functions, \(K_n = E_n/\hbar c, K_0 = cm_0/\hbar, \beta = (\kappa/4\pi)h/m_0\).

Condition (9) defines \(r_n\), whereas equations (10) through (12) are the boundary conditions and the normalization condition for the
function $f$, respectively. Condition (9) in the general case has the form $R(K, r_n) = R(G, r_n)$. Neglecting the proper gravitational field with the constant $G$, we shall write down this condition as $R(K, r_n) = 0$, to which equality (9) actually corresponds.

The right-hand sides of equations (7)-(8) are calculated basing on the general expression for the energy-momentum tensor of the complex scalar field:

$$T_{\mu\nu} = \Psi_{,\mu}^{+} \Psi_{,\nu} + \Psi_{,\nu}^{+} \Psi_{,\mu} - (\Psi_{,\mu}^{+} \Psi_{,\mu} - K_0^2 \Psi^{+} \Psi).$$  \hspace{1cm} (17)

The appropriate components $T_{\mu\nu}$ are obtained by summation over the index $m$ with application of characteristic identities for spherical functions [5] after the substitution of $\Psi = f(r) Y_{lm}(\theta, \varphi) \exp\left(-\frac{i E T}{\hbar}\right)$ into (17).

Even in the simplest approximation the problem of the stationary states of an elementary source in the proper gravitational field is a complicated mathematical problem. It becomes simpler if we confine ourselves to estimating only the energy spectrum. Equation (6) can be reduced in many ways to the equations [6]

$$f' = f P(r) + Q(r) z, \quad z' = f F(r) + S(r) z.$$  \hspace{1cm} (18)

This transition implies specific choice of $P, Q, F, S$, such that the conditions

$$P + S + Q'/Q + g = 0, \quad F Q + P' + P^2 + P g + h = 0$$  \hspace{1cm} (19)

should be fulfilled, where $g$ and $h$ correspond to equation (6) written in the form: $f'' + g f' + h f = Q$. Conditions (19) are satisfied, in particular, by $P, Q, F, S$ written as follows:

$$Q = 1, \quad P = S = -g/2, \quad F = \frac{1}{2} g' + \frac{1}{4} g^2 - h.$$  \hspace{1cm} (20)

Solutions of set (18) will be the functions: [6]

$$f = C \rho(r) \sin \theta(r), \quad c = C \rho(r) \cos \theta(r),$$  \hspace{1cm} (21)

where $C$ is an arbitrary constant, $\theta(r)$ is the solution of the equation:

$$\theta' = Q \cos^2 \theta + (P - S) \sin \theta \cos \theta - F \sin^2 \theta.$$  \hspace{1cm} (22)
and \( \rho(r) \) is found from the formula
\[
\rho(r) = \exp \int_0^r [P \sin^2 \theta + (Q + F) \sin \theta \cos \theta + S \cos^2 \theta] dr
\] (23)

In this case, the form of presentation of the solution in polar coordinates makes it possible to determine zeros of the functions \( f(r) \) at \( r = r_n \) with corresponding values of \( \theta = n\pi \) (\( n \) being an integer).

As one of the simplest approximations for \( \nu, \lambda \) we shall choose the dependence:
\[
e^-\lambda = 1 - \frac{\tilde{r}_n}{r + C_1} + \Lambda(r - C_2)^2 + C_3 r,
\] (24)

where
\[
\tilde{r}_n = \frac{2Kn_m}{c^2} = \frac{2KE_n}{c^4} = \left( \frac{2K\hbar}{c^3} \right) K_n, \quad C_1 = \frac{\tilde{r}_n}{\Lambda r_n^2}, \quad C_2 = r_n,
\]
\[
C_3 = \frac{\tilde{r} - n}{r_n(r_n + C_1)}.
\]

Earlier the estimate for \( K \) was adopted to be \( K \approx 1.7 \times 10^{29} Nm^2kg^{-2} \). If we assume that the observed value of the electron rest mass \( m_1 \) is its mass in the ground stationary state in the proper gravitational field, then \( m_o = m_1/3 \). From dimensionality considerations it follows that energy in the bound state is defined by the expression
\[
\left( \sqrt{Km_0} \right)^2 / r_1 = 0.17 \times 10^6 \times 1.6 \times 10^{-19} J,
\]
where \( r_1 \) is the classical electron radius. This leads to the estimate \( K \approx 5.1 \times 10^{31} Nm^2kg^{-2} \) which is later adopted as the starting one. It is known that by use of the dependence \( E_0 = \frac{e^2}{r_0} = mc^2[7] \) for the impulse we receive the expression \( P_i = \frac{4}{3} \frac{E_0}{c^2} v_i \), that is differentiated by the multiplier 4/3 from the correct expression for the impulse of the particles, the mass of which is \( m = \frac{E_0}{c^2} \). It is this fact that points at the correctness of the estimates received for the electron, because the lacking part of the energy is in the bound state. Discrepancies in the estimates \( K \) obtained by different methods are quite admissible, all the more so since their character is not catastrophic. From the condition that \( \mu \) is the electron energy density it follows: \( \mu = 1.1 \times 10^{30}J/m^3 \).
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\[ \Lambda = \kappa \mu = 4.4 \times 10^{29} \text{m}^{-2}. \] 

From (22) (with the equation for \( f(r) \) taken into account) it follows:

\[ 2\theta' = (1 + F) + (1 + F)\cos 2\theta \approx (1 - F), \tag{25} \]

where

\[ F = \frac{1}{2} \tilde{g}' + \frac{1}{4} \tilde{g}^2 - \tilde{h}, \quad \tilde{g} = r_n \left( \frac{2}{r} + \frac{\nu' - \lambda'}{2} \right), \]

\[ \tilde{h} = r_n^2 e^\lambda \left( K_n^2 e^{-\nu} - K_0^2 - \frac{l(l + 1)}{r^2} \right). \]

The integration of equation (25) and substitution of \( \theta = \pi n, r = r_n \) give the relation between \( K_n \) and \( r_n \):

\[ -2\pi n = -\frac{7}{4} - \frac{r_n K_n^2}{\Lambda^2} \sum_{i=1}^{3} \left\{ A_i \left[ \frac{(r_n + \alpha_i)^2}{2} - 2\alpha_i (r_n + \alpha_i) + \frac{\alpha^3}{(r_n + \alpha_i)} + r_n l(l + 1) \right] \right\} \]

\[ + 2C_1 (r_n + \alpha_i) + 2C_1 \frac{\alpha_i^2}{r_n + \alpha_i} + \frac{C_2^2 \alpha_i}{r_n + \alpha_i} \right] + B_i \left[ 9r_n + \alpha_i \right] + \frac{\alpha^2}{r_n + \alpha_i} + \frac{2C_1 \alpha_i}{r_n + \alpha_i} - \frac{C_2^2}{r_n + \alpha_i} \}

\[ \times \left[ d_1 r_n - \frac{C_1 d_2}{r_n} + \sum_{i=1}^{3} a_i (r_n + \alpha_i) \right] - \frac{K_n^2 r_n}{\Lambda^2} \left\{ \sum_{i=1}^{3} \left[ 2\alpha_i^2 A_i - 2\alpha_i B_i + \frac{K_0^2 \Lambda A_i}{K_n^2} (\alpha_i - C_1) + r_n^2 \Lambda (l + 1) a_i (C_1 - \alpha_i) \ln (r_n + \alpha_i) - r_n \Lambda^{-1} l(l + 1) (d_2 + C_1 d_1) \ln r_n \right] \right\} \tag{26} \]

The coefficients entering into equation (26) are coefficients at simple fractions in the expansion of polynomials, required for the integration, wherein \( \alpha_i \sim K_n, d_2 \sim A_i \sim r_n^{-5}, B_i \sim r_n^{-4}, A_i' \sim r_n^{-2}, a_i \sim r_n^{-4}, d_1 = r_n^{-4} \). For eliminating \( r_n \) from (26), there exists condition (9) (or the condition \( \exp \nu(K, r_n) = 1 \) equivalent to it for the approximation employed), but its direct use will complicate the already cumbersome expression (26) still further. At the same time, it easy to note that \( r_n \sim 10^{-3} r_{nc} \), where \( r_{nc} \) is
the Compton wavelength of a particle of the mass $m_n$, and, hence, $r_n \sim 10^{-3} K_n^{-1}$. The relation (26) per se is rather approximate, but, nevertheless, its availability, irrespective of the accuracy of the approximation, implies the existence of an energy spectrum as a consequence of the particle self-interaction with its own gravitational field in the range $r \leq r_n$, where mutually compensating action of the field and the particle takes place. With $l = 0$ the approximate solution (26), with the relation between $r_n$ and $K_n$ taken into account, has the form

$$E_n = E_0 \left(1 + \alpha e^{-\beta n}\right)^{-1},$$

(27)

where $\alpha = 1.65$, $\beta = 1.60$.

The relation (27) is concretized, proceeding from the assumption that the observed value of the electron rest mass is the value of its mass in the grounds stationary state in the proper gravitational field, the values $r_1 = 2.82 \times 10^{-15}$ m, $K_1 = 0.41 \times 10^{12}$ m$^{-1}$ giving exact zero of the function by the very definition of the numerical values for $r_n$ and $K$.

So, the presented numerical estimates for the electron show that within the range of its localization, with $K \sim 10^{31}$ N m$^{-2}$ and $\Lambda \sim 10^{29}$ m$^{-2}$, there exists the spectrum of stationary states in the proper gravitational field. The numerical value of $K$ is, certainly, universal for any elementary source. Existence of such numerical value $\Lambda$ denotes a phenomenon having a deep physical sense: introduction into density of the Lagrange function of a constant member independent on a state of the field. This means that the time-space has an inherent curving which is connected with neither the matter nor the gravitational waves. The distance at which the gravitational field with the constant is localized is less than the Compton wavelength, and for the electron, for example, this value is of the order of its classical radius. At distances larger than this one, the gravitational field is characterized by the constant $G$, i.e., correct transition to Classical GTR holds.

From equation (27) there follow in a rough approximation the numerical values of the stationary state energies: $E_1 = 0.511$ MeV, $E_2 = 0.638$ MeV, ..., $E_\infty = 0.681$ MeV.

The existence of stationary states in own gravitational field also completely corresponds the special relativity theory. According to SRT, relativistic link between energy and impulse is broken, if we
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assume that full electron’s energy is defined only by Lawrence’s electromagnetic energy [7]. If to expand the situation it is as follows [7]. Energy and impulse of the moving electron (with the assumption that the distribution of the electric charge is spherically symmetrical) is defined by expression:

\[
\bar{P} = \bar{V} \frac{\frac{4}{3} E_0 / c^2}{\sqrt{1 - \beta^2}}
\]

\[
E = \frac{E_0(1 + \frac{1}{3} V^2 / c^2)}{\sqrt{1 - \beta^2}}
\]

(28)

(29)

If these expressions were at the same time defining full impulse and full energy, the following relation would take place:

\[
E = \int (\bar{V} \frac{d\bar{P}}{dt}) dt
\]

(30)

However this relation cannot take place as integral in the right part equals to:

\[
\frac{\frac{4}{3} E_0}{\sqrt{1 - \beta^2}} + \text{const}
\]

(31)

If we find that impulse contrary to the energy has strictly electromagnetic character, then to \(E'\) of the moving and full energy \(E'_0\) of the resting electron, and also to rest mass \(E_0\), following relators will take place:

\[
E' = \frac{E'_0}{\sqrt{1 - \beta^2}}, \quad E'_0 = \frac{4}{3} E_0, \quad m_0 = \frac{E'_0}{c^2} = \frac{4}{3} \frac{E_0}{c^2},
\]

(32)

where rest mass \(m_0\) is defined by following expression:

\[
\bar{P} = \frac{m_0 \bar{V}}{\sqrt{1 - \beta^2}}
\]

(33)

Then from eq.(33) it follows that full energy of resting electron equals to \(\frac{4}{3}\) of its Lawrence’s electromagnetic energy. Numeric data of the electron’s stationary states spectrum in own gravitational field fully correspond to it.
Quantum transitions over stationary states must lead to the gravitational emission characterized by the constant $K$ with transition energies starting from 127 keV to 170 keV. Two circumstances are essential here.

**First.** The correspondence between the electromagnetic and gravitational interaction takes place on replacement of the electric charge $e$ by the gravitational "charge" $m\sqrt{K}$, so that the numerical values $K$ place the electromagnetic and gravitational emission effects on the same level (for instance, the electromagnetic and gravitational bremsstrahlung cross-sections will differ only by the factor 0.16 in the region of coincidence of the emission spectra).

**Second.** The natural width of the energy levels in the above-indicated spectrum of the electron stationary states will be very small. The small value of the energy level widths, compared to the electron energy spread in real conditions, explains why the gravitational emission effects are not observed as a mass phenomenon in epiphenomena, e.g., in the processes of electron beam bremsstrahlung on targets. However, there is a possibility of registration of gravitational emission spectrum lines. The results are given below. There is certain analytic interest in $\beta$-decay processes with asymmetry of emitted electrons [8], due to (as it is supposed to be) parity violation in weak interactions. $\beta$-asymmetry in angular distribution of electrons was registered for the first time during experiments with polarized nucleuses $^{27}_{60}$, $\beta$-spectrum of which is characterized by energies of MeV. If in the process of $\beta$-decay exited electrons are born, then along with decay scheme

$$n \rightarrow p + e^- + \bar{\nu}$$  \hspace{1cm} (34)

there will be also decay scheme

$$n \rightarrow p + (e^*)^- + \bar{\nu} \rightarrow e^- + \tilde{\gamma} + \bar{\nu},$$  \hspace{1cm} (35)

where $\tilde{\gamma}$ is the graviton.

Decay (35) is energetically limited by energy values of 1 MeV order (in rough approximation), taking into consideration that the difference between lower excitation level of electron’s energy (in own gravitational field) and general $<100$ keV and the very character $\beta$-spectrum. Consequently, $^{27}_{60}$ nucleuses decay can proceed with equal
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probability as it is described in scheme (34) or in scheme (35). For the light nucleuses, such as $^1_3\beta$-decay can only proceed as it is described in scheme (34). At the same time, emission of graviton by electron in magnetic field can be exactly the reason for $\beta$-asymmetry in angular distribution of electrons. If so, then the phenomenon of $\beta$-asymmetry will not be observed in light $\beta$-radioactive nucleuses. This would mean that $\beta$-asymmetry in angular distribution of electrons, which is interpreted as parity violation, is the result of electron’s gravitational emission, which should be manifested in existence of lower border $\beta$-decay, as that’s where $\beta$-asymmetry appears to be.

2 Gravitational Emission in Dense High Temperature Plasma

2.1 Excitation of Gravitational Emission in Plasma

For the above-indicated energies of transitions over stationary states in the own field and the energy level widths, the sole object in which gravitational emission can be realized as a mass phenomenon will be, as follows from the estimates given below, a dense high-temperature plasma.

Using the Born approximation for the bremsstrahlung cross-section, we can write down the expression for the electromagnetic bremsstrahlung per unit of volume per unit of time as

$$Q_e = \frac{32}{3} \frac{z^2 r_0^2}{137} mc^2 n_e n_i \frac{\sqrt{2kT_e}}{\pi m} = 0.17 \times 10^{-39} z^2 n_e n_i \sqrt{T_e}, \quad (36)$$

where $T_e$, $k$, $n_i$, $n_e$, $m$, $z$, $r_o$ are the electron temperature, Boltzmann’s constant, the concentration of the ionic and electronic components, the electron mass, the serial number of the ionic component, the classical electron radius, respectively.

Replacing $r_o$ by $r_g = 2K m/c^2$ (which corresponds to replacing the electric charge $e$ by the gravitational charge $m\sqrt{K}$, we can use for the gravitational bremsstrahlung the relation

$$Q_g = 0.16 Q_e. \quad (37)$$

From eq.(8) it follows that in a dense high-temperature plasma with parameters $n_e = n_i = 10^{23}$ m$^{-3}$, $T_e = 10^7$K, the specific power of
the electromagnetic bremsstrahlung is equal to \( \approx 0.53 \times 10^{10} \text{ J/m}^3 \text{ s} \), and the specific power of the gravitational bremsstrahlung is \( 0.86 \times 10^9 \text{ J/m}^3 \text{ s} \). These values of the plasma parameters, apparently, can be adopted as guide threshold values of an appreciable gravitational emission level, because the relative proportion of the electrons whose energy on the order of the energy of transitions in the own gravitational field, diminishes in accordance with the Maxwellian distribution exponent as \( T_e \) decreases.

### 2.2 Amplification of Gravitational Emission in Plasma

For the numerical values of the plasma parameters \( T_e = T_i = (10^7 - 10^8) \text{ K} \), \( n_e = n_i = (10^{23} - 10^{25}) \text{ m}^{-3} \) the electromagnetic bremsstrahlung spectrum will not change essentially with Compton scattering of electron emission, and the bremsstrahlung itself is a source of emission losses of a high-temperature plasma. The frequencies of this continuous spectrum are on the order of \( (10^{18} - 10^{20}) \text{ s}^{-1} \), while the plasma frequency for the above-cited plasma parameters is \( (10^{13} - 10^{14}) \text{ s}^{-1} \), or 0.1 eV of the energy of emitted quanta.

The fundamental distinction of the gravitational bremsstrahlung from the electromagnetic bremsstrahlung is the banded spectrum of the gravitational emission, corresponding to the spectrum of the electron stationary states in the own gravitational field.

The presence of cascade transitions from the upper excited levels to the lower ones will lead to that the electrons, becoming excited in the energy region above 100 keV, will be emitted, mainly, in the eV region, i.e., energy transfer along the spectrum to the low-frequency region will take place. Such energy transfer mechanism can take place only in quenching spontaneous emission from the lower electron energy levels in the own gravitational field, which rules out emission with quantum energy in the keV region. A detailed description of the mechanism of energy transfer along the spectrum will hereafter give its precise numerical characteristics. Nevertheless, undoubtedly, the very fact of its existence, conditioned by the banded character of the spectrum of the gravitational bremsstrahlung, can be asserted. The low-frequency character of the gravitational bremsstrahlung spectrum will lead to its amplification in plasma by virtue of the locking condition \( \omega_g \leq 0.5\sqrt{10^3 n_e} \) being fulfilled.
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From the standpoint of practical realization of the states of a high-temperature plasma compressed by the emitted gravitational field, two circumstances are of importance.

First. Plasma must comprise two components, with multiply charged ions added to hydrogen, these ions being necessary for quenching spontaneous emission of electrons from the ground energy levels in the own gravitational field. For this purpose it is necessary to have ions with the energy levels of electrons close to the energy levels of free excited electrons. Quenching of the lower excited states of the electrons will be particularly effective in the presence of a resonance between the energy of excited electron and the energy of electron excitation in the ion (in the limit, most favorable case — ionization energy). An increase of $z$ increases also the specific power of the gravitational bremsstrahlung, so that on the condition $\omega_g \leq 0.5\sqrt{10^3n_e}$ being fulfilled, the equality of the gas-kinetic pressure and the radiation pressure

$$k(n_eT_e + n_iT_i) = 0.16(0.1710^{-39}z^2n_en_i\sqrt{T_e})\Delta t$$

will take place at

$$\Delta t = (10^{-6} - 10^{-7})s$$

for the permissible parameter values of compressed plasma $n_e = (1 + a) n_i = (10^{25} - 10^{26}) m^{-3}$, $a > 2$, $T_e \approx T_e = 10^8 K$, $z > 10$.

Second. The necessity of plasma ejection from the region of the magnetic field with the tentative parameters $n_e = (10^{23} - 10^{24}) m^{-3}$, $T_e = (10^7 - 10^8) K$ with subsequent energy pumping from the magnetic field region.

2.3 A series of actions required for obtaining steady states of dense-high temperature plasma

- Forming and accelerating binary plasma with multivalent ions by accelerating magnetic field in a pulse high-current discharge.

- Injection of binary plasma from the space of the accelerating magnetic field:

exciting stationary states of an electron in its own gravitational field in the range of energy up to 171 keV with following radiation (Fig.
1) under the condition of quenching lower excited energy levels of ion electron shell of a heavy component (Fig. 2, including quenching excited state of electrons directly in nuclei of small sequential number as carbon) when retarding plasma bunch ejected from the space of the accelerating magnetic field. Cascade transitions from the upper levels are realized in the process of gravitational radiation energy transit to long-wave range.

The sequence of the operations is carried out in a two-sectional chamber (Fig. 3); the structure of the installation is most suitable for the claimed method of forming steady states of the dense high-temperature plasma [9]) with magnetodynamic outflow of plasma and further conversion of the plasma bunch energy (in the process of quenching) in the plasma heat energy for securing both further plasma heating and exciting gravitational radiation and its transit into a long-wave part of the spectrum with consequent plasma compression in the condition of radiation blocking and increasing.

Figure 1: Graviton emission when quenching an electron in a nucleus.

Of interest there are two modes of the installation operations depending on the work gas composition:

- a composition with hydrogen and xenon providing only for achieving steady states of plasma with consequent realization of thermonuclear reactions for compositions of (d+t) + multi-charge atoms type;

- a composition with hydrogen and carbon providing thermonuclear reactions of carbon cycle in plasma steady state mode, including energy pick-up in the form of electromagnetic radiation energy.
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Figure 2: Quenching lower excited states of electron by: a) many-electron ions (photoelectric effect with release of one electron or autoionization (Auger effect) with release of two electrons depending on the ion number and quenching energy); b) nuclei without electron shells when an excited electron returns to normal state transferring excess energy directly to the nucleus with higher probability for the lower energy levels of excited electrons.
3 Experimental data

3.1 Registration of electron gravitational radiation lines and energy spectrum in their own gravitational field:

It is known that the form of free neutron decay $\beta$-spectrum satisfactorily corroborates theoretical dependence for allowed transitions except soft parts of $\beta$-spectrum. Corresponding theoretical and experimental spectra are shown in Figs. 4, 5. The soft part of the spectrum is clearly linear exactly corresponding (taking into account kinetic energy of an outgoing electron) to the spectrum of electron steady states in its own gravitational field in the range of the steady state energies up to 171 keV.

In independent experiments [12] when at the same time electron energy distribution after electron beam passing through a foil was registered, clearly line energy spectrum was observed: Fig. 6(a). The line radiation spectrum is also clearly seen: Fig. 6(b) which cannot be explained only by the presence of accelerated electron groups. The quantitative identification of the spectrum requires more precise and broad measurements including identification algorithm of energy spectrum quantitative values relating directly to steady...
Figure 4: Beta-spectrum of free neutron decay obtained by Robson [10]. The straight line is Fermi graph, the experimental data points according to Robson [10].

Figure 5: Beta-spectrum of free neutron decay obtained by Christensen et al. [11]. The curve corresponds to a theoretical spectrum corrected for spectrometer energy resolution.
states of electrons. Nevertheless, registered the line type of electron energy spectrum and corresponding line electron radiation spectrum preliminary corroborate as a rough approximation the very fact of electron steady states in their own gravitational field exactly in the energy range up to 171 keV.

Figure 6: Energy distribution of (a) electrons and (b) X-ray quanta [12]

It is obvious that these data need to be supplemented with direct experimental identification both regarding both electron gravitational radiation spectrum lines and electron steady state energy spectrum
in its own gravitational field. Fig. 7 shows electron beam energy spectra in a pulse accelerator measured by a semicircular magnetic spectrometer. Two peaks of the energy spectra are connected to the feature of the pulse accelerator operations, the secondary pulse is due to lower voltage. This leads to the second (low-energy) maximum of the energy spectrum distribution.

A telemetry error in the middle and soft parts of the spectrum is not more than ± 2%. The magnetic spectrometer was used for measuring the energy spectrum of electrons after passing through the accelerator anode grid and also spectra of electrons after passing though a foil arranged behind the accelerator mesh anode. These data (and the calculated spectrum) are presented in Fig. 7. Similar measurements were carried out for Ti foil (foil thickness 50 µm) and Ta (foil thickness 10 µm). In case of Ti the measurements were limited from the top by energy of 0.148 MeV, and in case of TA by energy of 0.168 MeV. Above these values the measurement errors increase substantially (for this type of the accelerator). The difference between the normalized spectral densities of theoretical and experimental electron spectra after passing through Ti, Ta and Al foils is shown on Fig.9. The data indicate that there is a spectrum of electron energy states in their own gravitational field when the electrons are excited when passing through a foil. The obtained data are not sufficient for numerical spectrum identification but the very fact of the spectrum presence according to the data is doubtless.

3.2 Micropinch plasma electron gravitational radiation in pulse high-current discharges

The concept of a thermonuclear reactor on the principle of compressing dense high-temperature plasma by emitted gravitational field is supported by the processes of micropinching multicharged ion plasma in pulse high-current diodes. Figs 9, 10(a) show characteristic parts of micropinch soft X-ray radiation spectrum. Micropinch spectrum line widening does not correspond to existing electromagnetic conceptions but corresponds to such plasma thermodynamic states which can only be obtained with the help of compression by gravitational field, radiation flashes of which takes place during plasma thermalization in a discharge local space. Such statement is based on
Figure 7: Electron energy spectra: 1 – after passing the grid, 2 – after passing the Al foil 13 µm thick; 3 – spectrum calculation according to ELIZA program based on the database [13] for each spectrum. The spectrum is normalized by the standard.

Figure 8: Difference of spectral density for theoretical and experimental spectra of electrons passed through Ti, Ta and Al foils.
the comparison of experimental and expected parts of the spectrum shown in Fig. 10(a,b). Adjustment of the expected spectrum portion to the experimental one [15] was made by selecting average values of density \( \rho \), electron temperature \( T_e \) and velocity gradient \( U \) of the substance hydrodynamic motion.

As a mechanism of spectrum lines widening, a Doppler, radiation and impact widening were considered. Such adjustment according to said widening mechanisms does not lead to complete reproduction of the registered part of the micropinch radiation spectrum. This is the evidence (under the condition of independent conformation of the macroscopic parameters adjustment) of additional widening mechanism existence due to electron excited states and corresponding gravitational radiation spectrum part already not having clearly expressed lines because of energy transfer in the spectrum to the long-wave area.

That is to say that the additional mechanism of spectral lines widening of the characteristic electromagnetic radiation of multiple-charge ions (in the conditions of plasma compression by radiated gravitational field) is the only and unequivocal way of quenching electrons excited states at the radiating energy levels of ions and exciting these levels by gravitational radiation at resonance frequencies. *Such increase in probability of ion transitions in other states results in additional spectral lines widening of the characteristic radiation.* The reason for quick degradation of micropinches in various pulse high-currency discharges with multiple-charge ions is also clear. There is only partial thermolization of accelerated plasma with the power of gravitational radiation not sufficient for maintaining steady states.

The firm line in variant (b) corresponds to density of 0.1 g/cm\(^3\), the dotted line – to 0.01 g/cm\(^3\); it was assumed that \( T_e = 0.35 \) keV, [15].

It should be noted that widening of the spectral characteristic of the electromagnetic radiation [16] gave rise to a wrong conclusion about abnormal increase of the energy conservation law, while the widening of the spectral lines has a distinct and explicable nature, as stated above.
Figure 9: A part of vacuum sparkle spectrum and a corresponding part of solar flare spectrum. [14].
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Figure 10: Experimental (a) and calculated (b) parts of a micropinch spectrum normalized for line Ly$\beta$ intensity in the area of the basic state ionization threshold of He-like ions.
3.3 Thermonuclear plasma steady states generation

Available experimental data show that they can be reproduced in an active experiment directly with thermonuclear plasma on the ground of existence of gravitational radiation narrow-band spectrum in the range up to 171 keV with long-wave spectrum part realized by cascade electron transitions from the upper energy levels. Quenching lower excited states of electrons on the electron shell energy levels of heavy component ions in combination with cascade transitions will result in plasma compression in the conditions of blockage and gravitational radiation intensification.

Operation capacity of the chamber according to the scheme shown on Fig.3 with deuterium-tritium composition was tested experimentally. The obtained experimental data of plasma compression in the chamber [17]; however the holding time is not sufficient, there need to be longer holding time. The choice of such design as a design for a thermonuclear reactor is unequivocal since it is completely corresponds to the system of exciting and amplifying gravitational radiation when plasma is thermolized after outflow from the nozzle, and required additional compression actually takes place when the working plasma composition is changed (many-electron ions) and current-voltage characteristic of the charge changes correspondingly.

References

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Comment on
GRAVITATIONAL INTERACTION OF QUANTUM LEVEL AND CONSEQUENCES THEREOF

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The article poses in a new way the problem of the self energy of elementary particle (electron) when taking into account the gravitational field connected with this particle. Namely, the authors consider the energy levels of an electron in its own gravitational field. Assuming that at short distances the gravitational field is characterized by an extremely large gravitational constant $G$, the authors arrive at the spectrum of stationary quantum states which can be occupied by an electron. It is worth reminding that usually the field (electromagnetic, gravitational, and so on) generated by an elementary particle is expected to contribute to physical characteristics of the particle, specifically, to its mass, spin, electric and magnetic moments and so on.

It is this new point of view on the elementary particle self energy that forms the rational kernel of this article. Certainly, the authors justification of their assumption concerning the value of $G$ is
Comment

not convincing. However this hypothesis is non physically absurd because one usually believes that the gravitational interaction should be strong at the Planck scales.

The attempts undertaken by the authors to apply their idea to explanation of anomaly events in plasma are not substantiated. Nevertheless these attempts play also a positive role because they draw, once more, the attention to these anomalies and point out to the necessity of their explanation.

Taking into account all the above mentioned, one can infer that the paper at hand meets completely the basic goals of the Journal "The Old and New Concepts of Physics. An Open Dialogue Journal", namely, the papers submitted to this Journal should deal "with old and new ideas on Nature. The articles may be, and often must be, controversial, and occasionally hard to accept for other scientific journals".

In the first place the ideas themselves are put instead of their thorough elaboration. Thus the article "Gravitational interaction of quantum level and consequences thereof" by S.I. Fisenko and I.S. Fisenko can be recommended for publication in this Journal.
NEUTRINO CATALYSIS OF NUCLEAR SYNTHESIS REACTIONS IN COLD HYDROGEN

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Abstract

It is shown that the nuclear reaction of fusion in cold hydrogen is possible due to formation of metastable atoms of dineutronium existing as a bound state of two neutrons and one neutrino. Such atoms can appear in a reaction of deuterons with free or quasi-free electrons. The estimation of mass, size and lifetime of dineutronium atom is fulfilled.
1 Introduction

There are considered here low energy nuclear reactions (LENR) which lead to transmutations of elements. These reactions result at super-low energies of particles without accompanying intensive ionizing radiation [1]. The cold fusion (CF) is the nuclear synthesis reaction in gaseous, or absorbed by condensed matter hydrogen at the temperature $T_{cf} \leq 10^3 K$ that is essentially lower, than for thermonuclear reactions [2]. There is an opinion that physical lows forbid such processes. However, this opinion is wrong.

In 1937 L.W. Alvarez discovered the electron capture, what is the simplest example of LENR. In 1957 in the Berkley Nuclear Centre (USA), the research team headed by L.W. Alvarez [3] discovered the $\mu$ - catalysis. So, both LENR and the cold fusion were discovered by the same person, and he is the Nobel prize-winner (1968) L.W. Alvarez.

Unfortunately, majority of scientific community ignores the experimentally observable existence of LENR. This happens, to my mind, in consequence of:

- the absence of the conventional mechanism of the deuterons electrical charge screening;

- the fact, that the probability for thermal deuterons to overcome the Coulomb barrier is unimaginably small ($P \sim 10^{-2730}$).

A new mechanism of CF reactions were suggested in [1]. This mechanism does not contradict the known laws of physics and is based on the phenomenon of generating neutron-like particles with large internal energy. These particles were revealed in experiments with an electron accelerator [4]. Later on, these particles were interpreted as the bound state of the two neutrons and one neutrino [1].

Laws of physics do not impose basic theoretical bans on the existence of the metastable bound state of the two neutrons and neutrino, because a neutrino is a massive particle [5].

Due to interaction with quarks in a nucleon, a neutrino can "linger" inside it. This delay is caused, because the effective $N\nu$ - potential...
corresponding to $W$ - boson exchange (Fig. 1), is a short-range and very deep one. Its depth is still rather small to keep antineutrino, proton and electron in the bound state (i.e. like a neutron) for a long time, but just enough to consider a proton like the stable bound state of three particles, positron, neutron and neutrino. It is well known, that three-body effects allow an existence of 3 particles' bound states, which pair potentials are insufficiently deep to form 2 particles’ bound states.

A long lifetime of the neutrino inside a nucleus can be treated on the basis of exotic Miheev - Smirnov - Wolfenstein effect at low energies [8]. Let us explain this in more detail. If the energy of incoming electron is resonant (i.e. renormalized masses of all three types of neutrinos ($\nu_e, \nu_\mu, \nu_\tau$) inside a nucleon are approximately equal after the electron capture), the exotic nucleus is generated at the first stage of electroweak process (two left vertexes in the diagram 1), which cannot decay until an oscillation have been finished. The exotic nucleus $D_\nu$ is metastable, because the energy conservation law forbids its decay with $\mu$ - or $\tau$ - lepton emission. The channel $D_\nu \to 2n + \nu_e$ is also closed. Thus, theoretical consideration of the bound state of the neutrino inside a nucleus in the framework of any potential model gives us only phenomenological description of the observable effect.

From this standpoint, we shall consider hypothetical metastable exotic atom (exotic nucleus) dineutroneum, which is the bound state of two neutrons and one neutrino, as was mentioned above. The aim of this work is to estimate the mass, size and lifetime of the dineutroneum atom which is formed due to the interaction of deuterons with electrons.

Figure 1: The typical diagram of the electroweak process [6,7].
2 Main formalism

The known Hamiltonian of weak interaction is

\[ H' = \frac{G}{\sqrt{2}} \int J^{\lambda+}(\vec{r}) \hat{G}(\vec{r}, \vec{r}') J_\lambda(\vec{r}') d\vec{r} d\vec{r}', \] (1)

with \( G \) the Fermi constant of universal weak interaction, \( J_\lambda(\vec{r}) \) the weak current, and \( \hat{G}(\vec{r}, \vec{r}') \) the propagator. Let us introduce definition in accord to [9]

\[ J^{\lambda+} = (J_\lambda)^+, \quad \lambda = 1, 2, 3, \]
\[ J^{4+} = -(J_4)^+, \] (2)

and similarly for others 4- vector operators. In the standard model, the weak interaction is caused by exchange of the \( W^- \) boson with mass \( \approx 90 GeV \). Therefore, if we consider the low energy weak processes, an approximation \( m_W \to \infty \) can be used. Accordingly, the interaction is quite local, and components of the weak current in Hamiltonian (1) should be taken at the same point of space \( \hat{G}(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}') \).

Hence

\[ H' = \frac{G}{\sqrt{2}} \int J^{\lambda+}(\vec{r}) J_\lambda(\vec{r}) d\vec{r}. \] (3)

The Lorenz invariant weak current is well known. For example, \( \beta^- \) decay of a neutron is described by the Hamiltonian [9]

\[ H' = \frac{G}{\sqrt{2}} \int [\bar{\psi}_n(\vec{r}) \gamma^\lambda (1 + \gamma_5) \psi_p(\vec{r})]^+ \cdot [\bar{\psi}_e(\vec{r}) \gamma^\lambda (1 + \gamma_5) \psi_{\nu e}(\vec{r})] d\vec{r}. \] (4)

To describe the weak processes in nuclear physics, one needs a non-relativistic Hamiltonian \( h'(\vec{r}) \). The model of the Hamiltonian was derived in the early papers by Fermi, Gamov and Teller, and looks like [9]

\[ h'(\vec{r}, t) = \frac{G}{\sqrt{2}} \{ i \beta [f_1 \gamma_\lambda + f_2 \sigma_{\lambda \rho} k^\rho + (g_1 \gamma_\lambda + ig_2 k_\lambda) \gamma_5] \}^+ j^\lambda(\vec{r}, t) + h.c. \] (5)

In (5)

\[ j_\lambda(\vec{r}, t) = [i \bar{\psi}_l(\vec{r}) \gamma_\lambda (1 + \gamma_5) \psi_{\nu l}(\vec{r})] \cdot \exp \left( -\frac{i}{\hbar} (E_{\nu l} - E_l) t \right) \] (6)
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is the lepton current, \( E \) - the energy positive for particles and negative for antiparticles, \( f_1, f_2, g_1, g_2 \) the formfactors, \( \psi(\vec{r}) \) - lepton wave function (WF).

In the works devoted to the nuclear \( \beta \)-processes, the WFs of free leptons in (6) are usually chosen as plane waves with the momentum \( \vec{p} \). Thus, the lepton’s current (6) looks like:

\[
j_\lambda(\vec{r}, t) = L^{-3} b_\lambda \exp(i\vec{k} \cdot \vec{r}) \cdot \exp\left(-\frac{i}{\hbar} (E_\nu - E_e) t\right) \quad (7)
\]

where \( \vec{k} = \vec{\nu} - \vec{e} \) is the lepton transferred momentum, \( \nu \) the wave vector of the neutrino, \( e \) the wave vector of the electron, \( L^3 \) is the normalization volume,

\[
b_\lambda(m_e, m_\nu) = (i\bar{m}(m_e)\gamma_\lambda w_\nu(m_\nu)) \quad (7)
\]

and

\[
w_\nu(m_\nu) = (1 + \gamma_5) u_\nu(m_\nu). \quad (9)
\]

The spinor

\[
w_\nu(m_\nu) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} (1 - (\vec{\sigma} \cdot \vec{\nu})) \chi_{1/2}(m_\nu), \quad (10)
\]

\( m_\nu = \pm 1/2 \) the spin projection of neutrino (corresponds to spin ”up” and spin ”down”).

The lifetime of dineutroneum can be estimated within the approximation of allowed transitions. Therefore, we shall neglect the small contribution of the terms \( \hbar k/(Mc), p/Mc, kR \) due to the forbidden transitions, and obtain the non-relativistic limit of the Hamiltonian (5) in the plane wave approximation [9]:

\[
h'(\vec{r}) = \frac{G}{\sqrt{2}L^3} e^{i\vec{k} \cdot \vec{r}} \cdot \sum_{j=1}^{A} [i f_1 \cdot b_1 - g_1 (\vec{b} \cdot \vec{\sigma})]_j \cdot (\tau_+)_j \cdot \delta(\vec{r} - \vec{r}_j) + \ldots \quad (11)
\]

The Pauli matrixes \( \tau_1 \) and \( \tau_2 \) \( (\tau_{+1}, \tau_{-1}) \) are well known:

\[
\left\{ \begin{array}{c}
\tau_+ = (\tau_1 + i\tau_2)/2 = -\tau_{+1}/\sqrt{2} \quad \rightarrow \quad \tau_+|p> = |0>, \quad \tau_+|n> = |p>, \\
\tau_- = (\tau_1 - i\tau_2)/2 = \tau_{-1}/\sqrt{2} \quad \rightarrow \quad \tau_-|n> = |0>, \quad \tau_-|p> = |n>.
\end{array} \right. \quad (12)
\]

\(^1\)In reactions of electron capture, - decay into a bound state and in mesoatoms the charged lepton occupies the bound state and its WF belongs to the discrete spectrum.
The approximated Hamiltonian (11) is used to describe the nuclear processes with the dineutroneum.

First, we take into account, that the mass of dineutroneum is less than the double mass of the neutron. Therefore, neutrino in the atom of dineutroneum is in the bound state, and the Hamiltonian looks like

\[
h'(\vec{r}) = \frac{G_\beta}{\sqrt{2}L^{3/2}} \psi_\nu(\vec{r}_c) \cdot e^{-i\vec{c} \cdot \vec{r}} \cdot \left\{ \sum_{i=1}^{2} \delta(\vec{r} - \vec{r}_c) \left[ i\hbar \cdot (\vec{b}_i \cdot \vec{\sigma}_i) \right] \tau^{(i)}_+ \right\} + h.c,
\]

where \(\psi_\nu(\vec{r}_c)\) is the spatial part of the neutrino’s WF, \(G_\beta = f_1 G\), index \(c\) indicates the radius-vector of the neutrino which origin is in the centre-mass of the dineutroneum because of translation-invariance of the Hamiltonian \(h'(\vec{r})\).

According to a ”golden Fermi’s rule”, the probability of the transition to the continuum states per unit of time is equal:

\[
dw_{fi} = \frac{2\pi}{\hbar} \delta(E_f - E_i) \left| \langle f | V | i \rangle \right|^2 dn_f.
\]

Hence, the decay probability of the bound state of two neutrons and one neutrino within the channel \(D_\nu \rightarrow d + e^-\) per the time unit is equal to:

\[
w_{D_\nu \rightarrow d + e^-} = \frac{2\pi}{\hbar} \int \frac{L^3 d\vec{p}_e}{(2\pi\hbar)^3} \cdot \frac{L^3 d\vec{p}_d}{(2\pi\hbar)^3} \cdot \delta(E_i - E_f) \times
\]

\[
\times \int \left\langle \left| \left| d | h'(\vec{r}) | D^{(N)}_\nu \right| \right|^2 \right\rangle d\vec{r}.
\]

The WFs \(|D^{(N)}_\nu >\) and \(< d |\) depend on the coordinates, spins and isospins of nucleons, and matrix elements of the transition \(D_\nu \rightarrow d + e^-\) in the space of leptons are already included into the Hamiltonian \(h'(\vec{r})\) by definition. The external triangular brackets in (15) mean the averaging by projections of spins of all initial particles, and analogous summation in the final state.

Let us now consider the \(\beta\) - decay of the dineutroneum. The initial
and final states in this case are\(^2\):\(^{\quad 16}\)

\[
\begin{align*}
&D_{\nu}^{(N)} = \frac{1}{\sqrt{L^3}} e^{ik_{D\nu} \bar{R}_{D\nu} \psi_{2n}(\vec{r}_2 - \vec{r}_1) \chi_{00}(\vec{S}) \chi_{1-1}(\vec{T})}, \\
&d = \frac{1}{\sqrt{L^3}} e^{ik_d \bar{R}_d \psi_d(\vec{r}_2 - \vec{r}_1) \chi_{1m_d}(\vec{S}) \chi_{00}(\vec{T})}.
\end{align*}
\]

Consequently, the matrix element in (15) looks like

\[
\int \langle d | h'(\vec{r}') \rangle D_{\nu}^{(N)} d\vec{r}' = \frac{1}{L^3} \int d\vec{r}' d\vec{r}_1 d\vec{r}_2 e^{i(k_{D\nu} \bar{R}_{D\nu} - k_d \bar{R}_d) \psi_d^*(\vec{r}'')} \times \\
\psi_{2n}(\vec{r}'') \chi_{1m_d}(\vec{S}) \chi_{00}(\vec{T}) | h'(\vec{r}') | \chi_{00}(\vec{S}) \chi_{1-1}(\vec{T}) \rangle,
\]

where \(\vec{r}'' = \vec{r}_2 - \vec{r}_1\).

The "nuclear" spin of the dineutron \(J_i = 0\) and the deuteron's spin \(J_f = 1\). Thus, we deal with the Gamov - Teller transition. According to it

\[
h'_{GT}(\vec{r}) = -\frac{\lambda \cdot G_\beta}{\sqrt{2} L^{3/2}} \psi_\nu(\vec{r}_c) e^{-\bar{r}\cdot r} \left\{ \sum_{i=1}^2 \delta(\vec{r} - \vec{r}_i) \cdot (\vec{b} \cdot \vec{\sigma}_i) \cdot \tau_+^i \right\} + h.c.
\]

We consider the dineutron \(\beta\) - decay in its rest system. In this case \(k_{D\nu} = 0\), and (18) is simplified (details see in the Appendix):

\[
\int <d| h'(\vec{r}) | D_{\nu}^{(N)} > d\vec{r} = \frac{\lambda G_\beta \sqrt{3}}{2L^{9/2}} C_{1/2m_\nu}^{1/2} \bar{R}_{1/2m_d,1/2m_\nu} \times \\
\times \int d\vec{r}_1 d\vec{r}_2 e^{i\vec{k}_{D\nu} \bar{R}_{D\nu} \psi_d^*(\vec{r}) \psi_\nu(\vec{r})} \sum_{i=1}^2 \psi_\nu(\vec{r}_i - \vec{R}) e^{-i\bar{r}\cdot r_i}.
\]

We determine the formfactor

\[
f_{\text{overlap}}^{d \leftrightarrow D_{\nu}}(\vec{c}) = \int \cos(\vec{c} \cdot \vec{r}/2) \psi_d^*(\vec{r}) \psi_\nu(\vec{r}/2) \psi_{2n}(\vec{r}) d\vec{r} \equiv (V_{\text{eff}_D})^{-1/2}.
\]

The \(V_{\text{eff}_D}\) means an effective volume of exotic atom of dineutron. This circumstance allows to present eq. (19) in the extremely compact form:

\[
\int \langle d | h'(\vec{r}) | D_{\nu}^{(N)} > d\vec{r} = \]

\(^2\)See details in \([1]\).
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\[
= \frac{\lambda G_\beta \sqrt{2}}{L^{9/2}} (2\pi)^3 \delta(k_d + e) f_{\text{overlap}}^d D_\nu \bar{e} \left| -1 \right>^{1/2 + m_\nu} C^{1m_\nu}_{1/2 - m_e, 1/2} \cdot \tag{21}
\]

In turn, eq. (15) can be presented in the form which is suitable for numerical calculations

\[
w_{D_\nu \rightarrow d + e^-} = \frac{2\pi}{\hbar} \int \frac{dp_e}{(2\pi \hbar)^3} \cdot \delta(E_i - E_f) \cdot 3 \cdot |\lambda G_\beta f_{\text{overlap}}^d D_\nu \bar{e} \left| \cdot |\lambda G_\beta f_{\text{overlap}}^d D_\nu \bar{e} \right|^2,
\]

and evaluate the integral

\[
I_{D_\nu \rightarrow d + e^-}^{ph}(p_e) = \int dp_e \cdot \delta(E_i - E_f) = 4\pi \int dp_e p_e^2 \delta(E_{D_\nu} - E_d - E_e).
\]

All the particles in our case are non-relativistic. Consequently,

\[
E_{D_\nu} = m_{D_\nu} c^2 + \frac{p_{D_\nu}^2}{2m_{D_\nu}},
\]

\[
E_d = m_d c^2 + \frac{p_d^2}{2m_d},
\]

\[
E_e = m_e c^2 + \frac{p_e^2}{2m_e}.
\]

As a result,

\[
I_{D_\nu \rightarrow d + e^-}^{ph} \approx 4\pi p_e m_e,
\]

where the momentum

\[
p_e = \sqrt{2m_e (m_{D_\nu} c^2 - m_d c^2 - m_e c^2)}, \tag{26}
\]

corresponds to \( \vec{p}_{D_\nu} = 0 \) in the rest system of dineutroneum.

The internal energy of the dineutroneum \( U_{D_\nu} \) is equal to

\[
U_{D_\nu} = m_{D_\nu} c^2 - m_d c^2 - m_e c^2 > 0. \tag{27}
\]

Thus, eq. (26) can be presented in a rather compact form

\[
p_e = \sqrt{2m_e U_{D_\nu}} \tag{28}
\]

and we get the following expression:

\[
w_{D_\nu \rightarrow d + e^-} = \frac{3}{\pi \hbar^4} \cdot m_e \cdot \sqrt{2m_e U_{D_\nu}} \cdot |\lambda G_\beta f_{\text{overlap}}^d D_\nu \bar{e} \left| \cdot |\lambda G_\beta f_{\text{overlap}}^d D_\nu \bar{e} \right|^2. \tag{29}
\]
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The momentum dependence of the formfactor (20) at the low energies can be neglected

\[
f_{\text{overlap}}^{d\leftrightarrow D_\nu} = \int \psi_d^*(\vec{r})\psi_\nu(\vec{r}/2)\psi_{2n}(\vec{r})d\vec{r} \equiv (V_{\text{eff}}^{D_\nu})^{-1/2} \tag{30}
\]

and

\[
w_{D_\nu \rightarrow d+e^-} = \frac{3\lambda^2 \cdot G_\beta}{\pi \hbar^4 V_{\text{eff}}^{D_\nu}} c \dot{m}_e \cdot \sqrt{2m_e U_{D_\nu}}. \tag{31}
\]

Formula (30) determines the overlap integral \( f_{\text{overlap}}^{d\leftrightarrow D_\nu} \). For estimations, we accept that the bound particles participating in the reaction \( D_\nu \rightarrow d + e^- \) have the orbital momentum equal to zero, and their wave functions look like

\[
\psi_d(r) = \frac{1}{\sqrt{4\pi}} \frac{\chi_d(r)}{r}; \quad \psi_{2n} r \frac{1}{\sqrt{4\pi}} \frac{\chi_{2n}(r)}{r}; \quad \psi_\nu(r) = \frac{1}{\sqrt{4\pi}} \frac{\chi_\nu(r)}{r}. \tag{32}
\]

Only Hulten’s WF \( \chi_d(r) \) in (32) is known

\[
\chi_d(r) = A \exp(-\alpha_d r)[1 - \exp(-\mu r)] \tag{33}
\]

with the normalization constant

\[
A_d = [2\alpha_d(\alpha_d + \mu)(2\alpha + \mu)]^{1/2} \mu^{-1}. \tag{34}
\]

Here \( \alpha_d = \sqrt{m_N|E|/\hbar} \approx 0.232 \text{ fm}^{-1}, \quad \mu \approx 1.1 \text{ fm}^{-1} \) [10]. We assume that

\[
\chi_{2n}(r) = A_{2n} \exp(-\alpha_{2n} r)[1 - \exp(-\mu r)], \tag{35}
\]

with

\[
A_{2n} = [2\alpha_{2n}(\alpha_{2n} + \mu)(2\alpha_{2n} + \mu)]^{1/2} \mu^{-1} \tag{36}
\]

and equal parameters \( \mu \) for deuteron and dineutroneum. For the sake of simplicity we suppose

\[
\chi_\nu(r) = A_\nu \exp(-2\kappa r), \tag{37}
\]

where \( A_\nu = [4\kappa]^{1/2} \).

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According to (30)

$$f_{overlap}^d \leftrightarrow D^{(N)}_\nu = \frac{2}{\sqrt{4\pi}} \int_0^\infty \frac{\chi_d(r)\chi_\nu(r/2)\chi_{2n}(r)}{r} dr. \quad (38)$$

This integral in a view of (33), (35) and (37) can be calculated analytically

$$f_{overlap}^d \leftrightarrow D^{(N)}_\nu = A_{2n}A_dA_\nu \ln \left( \frac{\alpha_{2n}^{(\nu)}}{\alpha_{2n}^{(\nu)} - \mu^2} \right), \quad (39)$$

where $\alpha_{2n}^{(\nu)} = \kappa + \alpha_d + \alpha_{2n} + \mu$. In this work, we suppose $\chi_d(r) \approx \chi_{2n}(r)$ (i.e. $\alpha_{2n} \sim \alpha_d$).

Let us estimate $V_{eff}^{D_\nu}$ in the rough approximation $\alpha_{2n} = \alpha_d$. The decaying dineutroneum is created in the reaction of electron capture by deuteron. Thus, we suppose neutrino to be ”smeared” in a deuteron. This assumption implies an estimation $\kappa = \alpha_{2n} = 0.232 \text{ fm}^{-1}$. Consequently, we estimate $V_{eff}^{D_\nu} \approx 20 \text{ fm}^3$.

The standard Coulomb corrections also can be considered

$$w_{D_\nu \leftrightarrow d+e^-} = \frac{3|\lambda|^2 \cdot |G_\beta|^2}{\pi \hbar^4 V_{eff}^{D_\nu}} \cdot m_e \cdot p_e \cdot F(\eta). \quad (40)$$

The Fermi function $F(\eta)$ in the ”point-like deuteron” approximation is equal to [11]

$$F(\eta) \approx \pi \eta \cdot exp(\pi \eta) \cdot sh^{-1}(\pi \eta). \quad (41)$$

All previous calculations were carried out under the assumption, that neutrino inside the dineutroneum is the electron’s neutrino $|\nu_e \rangle$. Taking account the MSV- effect, we insert the electron’s neutrino weight $\langle | \nu | \nu_e > |^2 \rangle \sim \frac{1}{2} \sim \frac{1}{3}$ into (40) [8]:

$$w_{D_\nu \leftrightarrow d+e^-} = \langle | \nu | \nu_e > |^2 \rangle \cdot \frac{3|\lambda|^2 \cdot |G_\beta|^2}{\pi \hbar^4 V_{eff}^{D_\nu}} \cdot m_e \cdot p_e \cdot F(\eta), \quad (42)$$

where $\langle | \nu | \nu_e > |^2 \rangle$ is the probability for the neutrino to be in the state $|\nu_e \rangle$ in the dineutroneum.
Neutrino catalysis of nuclear synthesis reactions in cold hydrogen

<table>
<thead>
<tr>
<th>$T_e$ [eV]</th>
<th>$w^0_{D\nu \to d + e^-}$</th>
<th>$w^c_{D\nu \to d + e^-}$</th>
<th>$t^c_{D\nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>16.5</td>
<td>$1.1 \times 10^3$</td>
<td>$9.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>1.0</td>
<td>$4.8 \times 10^1$</td>
<td>$1.1 \times 10^3$</td>
<td>$9.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>10</td>
<td>$1.5 \times 10^2$</td>
<td>$1.1 \times 10^3$</td>
<td>$9.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>$10^2$</td>
<td>$4.8 \times 10^3$</td>
<td>$1.2 \times 10^3$</td>
<td>$8.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>$10^4$</td>
<td>$1.5 \times 10^3$</td>
<td>$2.1 \times 10^3$</td>
<td>$4.7 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 1: The energy dependence of $w^0$, $w^c$ and the lifetime $\tau^c_{D\nu}$.

In the table 1 the values of $w^0$, $w^c$ and a lifetime $\tau^c_{D\nu} = 1/w^c_{D\nu \leftrightarrow d+d^-}$ are displayed. An approximation $V^D_{e eff} = 20 \text{ fm}^3$, $<|\nu|\nu_e|> = 1$ is used.

We can see from the table 1, that at the low energies, the probability of the $\beta$ - decay of the dineutroneum can increase almost by two orders of magnitude owing to the Coulomb interaction. At $T_e > 1KeV$ this effect becomes insignificant. Therefore, if the dineutroneum atom is created, it lives long enough. The threshold of its creation is estimated at the level $10 - 15eV$, what is much lower than that for thermonuclear reactions $T_{thresh} << T_{tn} \sim 10KeV$.

Let us consider the dependence of the dineutroneum lifetime on its size. This dependence should be taking account, since the triplet length of the neutron-neutron scattering much exceeds the deuteron’s effective radius $r_d$. Table 2 demonstrates the results of theoretical calculations of the $\beta$ - decay rate $w^c_{D\nu \to d + e^-}$ and lifetime $\tau^c_{D\nu}$ as a function of the parameter $\alpha_d/\alpha_{2n}$ at $T_e = 10TeV$ (we suppose that $\kappa = \alpha_{2n}$).

It follows from Table 2, that if the size of dineutroneum alike the size of deuterium mesoatom, its lifetime would be almost 3 seconds. Consequently, one can conclude that the exotic dineutroneum atom is metastable and its lifetime $\tau_{D\nu} \sim 10^{-3} \text{ sec}$, i.e. three orders more than lifetime of the muon [5] $\tau_\mu = (2.197019 \pm 0.000021) \times 10^{-6} \text{ s}$.

Our preliminary analysis shows, that such properties of dineutroneum as: metastability, electrical neutrality and small sizes, allow nuclear reactions of dineutroneum with nuclei in condensed matter.

If we take into account large cross section of the $e$ - capture ($\sigma \sim 10 \text{ mbarn}$ for the $e^- + D \to D\nu + X$ reaction [4], it is possible easily explain a numerous experimental data on cold fusion in the condensed...
Table 2: The dependence of rate of the $\beta$ - decay of the dineutroneum on the ratio $\alpha_d/\alpha_{2n}$.

<table>
<thead>
<tr>
<th>$\alpha_d/\alpha_{2n}$</th>
<th>$w_{D_\nu \rightarrow d + e^-}^c$</th>
<th>$\tau_{D_\nu}^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1.1 \times 10^3$</td>
<td>$9.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>10</td>
<td>$3.1 \times 10^3$</td>
<td>$3.2 \times 10^{-2}$</td>
</tr>
<tr>
<td>$10^2$</td>
<td>$3.6 \times 10^{-1}$</td>
<td>2.7</td>
</tr>
</tbody>
</table>

matter (see [1, 12, 13, 19, 20]). For example, there are observed [19,20] such reactions as

$$D_\nu + \frac{A}{46} Pd \rightarrow \frac{A+1}{46} Pd + p + e^-$$

$$D_\nu + p \rightarrow \left\{ \begin{array}{c} t + \nu_e + 5.45 Mev \\ \frac{3}{2} He + e^- + 5.47 Mev \end{array} \right. \quad (43)$$

$$D_\nu + d \rightarrow \frac{4}{2} He + e^- + 23.85 MeV.$$

3 Conclusions:

1. The atom of dineutroneum is metastable ($\tau_\nu \sim 10^{-3} \text{s}$).
2. The size of dineutroneum are commensurable with the size of deuteron.
3. The mass of dineutroneum $M_{D_\nu} = 2.01410223 \ e = 1876.0979650 \ MeV$.
4. Metastability, electrical neutrality and small size allow nuclear reactions of the dineutroneum exotic atoms with nuclei both in gases, and in a condensed matter (for example: $D_\nu + p \rightarrow t + \nu_e$, $D_\nu + p \rightarrow \frac{3}{2} He + e^-$, $D_\nu + d \rightarrow \frac{4}{2} He + e^-$). This presents the clear explanation of many experiments on cold fusion [14-22].

Acknowledgments

Author would like to thank Dr. Yu.V. Popov and prof. Yu.B. Magarshak for fruitful discussions.
Appendix. Spin and isospins matrix elements

The isospins matrix element is equal
\[
\langle \chi^{00}(\vec{T})|\tau_-(i)\chi^{1-1}(\vec{T}) \rangle = \\
= \frac{1}{\sqrt{2}} \langle [p(1)n(2) - p(2)n(1)]|\tau_+(i)|n(1)n(2) \rangle = \frac{(-1)^{i-1}}{\sqrt{2}}.
\] (A.1)

The spin matrix element is more complicated
\[
\langle \chi^{1m_d}(\vec{S})|\sigma^{(i)}_k|\chi^{00}(\vec{S}) \rangle = \sum_{m_1 m_2} C^{1m_d}_{1/2m_1,1/2m_2} \sum_{m_3 m_4} C^{00}_{1/2m_3,1/2m_4} \times \\
\times \langle \chi^{(1)}_{1/2m_1}\chi^{(2)}_{1/2m_2}|\sigma^{(i)}_k|\chi^{(1)}_{1/2m_3}\chi^{(2)}_{1/2m_4} \rangle
\] (A.2)

According to the Clebsh - Gordan coefficients’ properties
\[
\langle \chi^{1m_d}(\vec{S})|\sigma^{(i)}_k|\chi^{00}(\vec{S}) \rangle = -\langle \chi^{1m_d}(\vec{S})|\sigma^{(i)}_k|\chi^{00}(\vec{S}) \rangle.
\] (A.3)

If \( i = 1 \) one obtains
\[
S_f = \sum_{m_1 m_2} C^{1m_d}_{1/2m_1,1/2m_2} \sum_{m_3 m_4} C^{00}_{1/2m_3,1/2m_4} \delta_{m_2 m_4} \langle \chi^{(1)}_{1/2m_1}|\sigma^{(1)}_k|\chi^{(1)}_{1/2m_3} \rangle.
\] (A.4)

It is evident, that
\[
\sigma|m u \chi_{1/2}\sigma = -\sqrt{3} \sum_{\sigma'} C^{1/2\sigma'}_{1,\mu,1/2}\chi_{1/2}\sigma'.
\] (A.5)

Thus
\[
S_f = \sqrt{3} \sum_{m_1, m_2, m_3} C^{1m_d}_{1/2m_1,1/2m_2} C^{00}_{1/2m_2,1/2m_1} C^{km_1}_{1,1/2m_1}
\] (A.6)

and:
\[
S_{C - G} = \sum_{m'', \sigma, \sigma'} C^{j' m'}_{j'' m'' 1/2 \sigma} C^{j m}_{j'' m'' 1/2 \sigma} C^{1/2\sigma'}_{1,\mu,1/2\sigma} \equiv \sqrt{2} \cdot \hat{j} \cdot \hat{j} \cdot F_{ang} =
\]
Yu. L. Ratis

\[ = \sqrt{2 \cdot \hat{j}' \cdot \hat{j}} \sum_{m'',\sigma,\sigma'} (-1)^{l-1/2+m''+j''-1/2+m+1/2+\sigma'} \times \]

\[ \times \left( \begin{array}{ccc} j'' & 1/2 & j' \\ m'' & \sigma' & -m' \end{array} \right) \left( \begin{array}{ccc} j'' & 1/2 & j \\ m'' & \sigma & -m \end{array} \right) \left( \begin{array}{ccc} 1 & 1/2 & 1/2 \\ \mu & \sigma & -\sigma' \end{array} \right), \]

where \( \hat{j} = 2j + 1 \). The sum of three \( 3jm \) - Wigner symbols \( F_{\text{ang}} \) is equal to

\[ F_{\text{ang}} = (-1)^{j-1/2+l+j'+m'} \left( \begin{array}{ccc} j' & 1 & j \\ m' & -\mu & m \end{array} \right) \left( \begin{array}{ccc} j' & 1 & j \\ 1/2 & l & 1/2 \end{array} \right). \]  

(A.7)

Inserting (8) in (7), we get the value of \( S \):

\[ S = \sqrt{2(2j+1)(-1)^j+l-1/2}C_{1\mu m}^{j'm'} \left( \begin{array}{ccc} j' & 1 & j \\ 1/2 & l & 1/2 \end{array} \right). \]  

(A.8)

Thus, we derive the result

\[ \left\langle \chi_{1m_d}(\vec{S})|\sigma_k^{(i)}|\chi_{00}(\vec{S}) \right\rangle = (-1)^{i-1}\delta_{-k,m_d}. \]  

(A.10)

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Comment on
NEUTRINO CATALYSIS OF NUCLEAR SYNTHESIS REACTIONS IN COLD HYDROGEN

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In the paper of Dr. Ratis it has been discussed how the nuclear reaction of fusion in cold hydrogen is possible due to formation of metastable atoms of dineutroneum existing as a bound state of two neutrons and one neutrino. According to this approach, such atoms can appear in a reaction of deuterons with free or quasi-free electrons.

I do not have any opinion concerning the possibility of cold fusion. As it is very well know, it is a very exciting field, without any experimental conformation so far. However, the main result, namely the formation of metastable atoms of dineutroneum existing as a bound state of two neutrons and one neutrino, is extremely interesting. This model and calculations might be important for understanding of a number of not solved, or partially solved, problems of atom theory and quantum chemistry. I strongly recommend this paper of Dr. Ratis for publication.
According to the classical special theory of relativity any nonstationary system moving with velocity $v$ must evolve (e.g., decay) $1/\gamma$ times slower than the system at rest, $\gamma = (1 - v^2)^{-1/2}$ (the Einstein retardation ER). Quantum mechanics allows one to calculate the evolution of both systems separately and to compare them thus verifying ER. It is shown here that ER is not valid for a simple system: the spreading packet of the free spinless particle. Earlier it was shown that ER does not hold for some other systems. So one may state that ER is not a universal kinematic law in quantum mechanics.
1 INTRODUCTION

Experiments show that moving unstable particles (e.g., $\mu$-mesons, $\pi$-mesons) decay slower than the particles at rest. More explicitly, let $N_0(t) = N \exp(-t/\tau_0)$ be the decay law of the particles at rest, $\tau_0$ being life-time. Then the life time of particles moving with velocity $\vec{v}$ is $\tau_v = \tau_0 \gamma$, $\gamma = (1 - v^2)^{-1/2}$, and the decay law of moving particles is

$$N_v(t) = N \exp(-t/\tau_v) = N \exp(-t/\tau_0 \gamma) = N_0(t/\gamma).$$  

(1)

One may rewrite Eq. (1) as

$$N_p(t) = N_0(t/\gamma), \quad \gamma = \sqrt{p^2 + m^2}/m$$  

(2)

using the corresponding momentum $\vec{p}$ instead of velocity $\vec{v}$: $\vec{p} = E \vec{v}$.

The usual theoretical explanation of Eqs. (1), (2) is based on the Einsteinian special relativity theory. It is set forth as follows. A moving clock has a slower course as compared with the clock at rest, namely $\gamma d\tau = dt$, e.g., see [1], Ch. 2, Eqs. (36) or (38). The unstable substance may serve as a clock, see [1], Ch. 2. Being the clock, the moving ensemble of unstable particles must decay slower than the ensemble at rest. This is described by Eq. (1): $N_v$ assumes at the moment $t$ the value which $N_0$ assumes at the moment $t/\gamma$.

This argumentation may be applied to any nonstationary physical system which may serve as a clock. Instead of $N_v(t)$ another time-dependent observable $F_v(t)$ may be considered. As the example, the dispersion $\sigma^2(t)$ of the spreading packet may be examined, see Sect. 2 below. In the same manner as above one may argue that the equation

$$F_v(t) = F_0(t/\gamma)$$  

(3)

must hold. Equation (1) is a particular case of Eq. (3). Eq. (3) means that $F_v$ assumes at the moment $t$ the value which $F_0$ assumes at the earlier moment $t/\gamma$. I call relation (3) Einsteinian retardation ER. It is a kinematic law in special relativity, see [1], Ch. 2.

However, clocks considered in special relativity are nonquantum objects: they have simultaneously a definite position (e.g., being in frame’s origin) and definite velocity (e.g., zero velocity), see the beginning of Ch. (2.6) in [1]. This is impossible for such quantum objects.
as $\mu$ or $\pi$ mesons. Therefore, the usual explanation of relations (1), (2), (3) is not valid for quantum objects.

However, one may verify the validity of these relations in quantum mechanics. The number of particles and other time-dependent observables may be considered as quantum observables. Using quantum mechanics one may calculate the observables separately for the moving system and the system at rest. Comparing them one may ascertain whether Eqs. (1), (2), (3) hold. For unstable particles this approach was considered in [2]-[6]. The result may be formulated as follows: ER does not hold exactly but it is valid up to high precision.

Oscillating systems ($K_0-\bar{K}_0$ mesons and oscillating neutrino) were considered in [5], [6]. It is shown that large deviations from ER may exist.

A moving nonstationary system was discussed in refs. [6], [7] which evolves faster than the system at rest: $F_v(t) = F_0(\gamma t)$ holds instead of Eq. (3)!

In this paper I consider in Sect. 2 the simple nonstationary system: the spreading wave packet of the free spinless particle. Packet dispersions (see below Eq. (4)) are used as the time-dependent observables which describe packet spreading. In Sect. 2 I calculate the longitudinal dispersion $\sigma^2_l(\vec{v}, t)$ (dispersion along the packet velocity $\vec{v} = \vec{p}/E$). It is compared in Sect. 3 with the dispersion $\sigma^2(0, t)$ of the packet at rest. The connection $\sigma^2_l(\vec{v}, t) = \sigma^2(0, t/\gamma^3)$ is obtained. So $\sigma^2_l(\vec{v}, t)$ is retarded as compared to $\sigma^2(0, t)$ but Eq. (3) is not valid, i.e. ER fails. The premises of the result are summed up in Sect. 3.

## 2 Dispersions of Gaussian packet

ER is the kinematic statement on the time evolution of nonstationary physical systems which may be considered as clocks. So the quantum mechanical consideration of ER must deal with time-dependent observables (so that the known $S$-matrix approach is not relevant).

In the capacity of the nonstationary system let us consider the spreading packet of the scalar particle. Let us consider the packet dispersions $\sigma^2_1(t)$, $\sigma^2_2(t)$, $\sigma^2_3(t)$

$$\sigma^2_j(t) = \int d^3x x^2_j \rho(\vec{x}, t) - \left[ \int d^3x \rho(\vec{x}, t) \right]^2, \quad j = 1, 2, 3$$

(4)
as time-dependent observables. Here \( \rho(\vec{x}, t) \) is the probability density to find the particle at the point \( \vec{x} \) at time \( t \). The density and dispersions may be experimentally measured. In quantum theory \( \rho \) is expressed in terms of the packet wave function \( \Psi \), the positive-energy solution of the Klein-Gordon equation

\[
\frac{i}{\hbar} \frac{\partial \Psi}{\partial t} = \hat{E} \Psi, \quad \hat{E} \equiv \left[ (-i\partial / \partial \vec{x})^2 + m^2 \right]^{1/2}.
\] (5)

The known usual expression of \( \rho \) is

\[
\rho(\vec{x}, t) \sim \hat{E} \Psi^*(\vec{x}, t) \Psi(\vec{x}, t) + \Psi^*(\vec{x}, t) \hat{E} \Psi(\vec{x}, t),\ e.g.\ see\ [8],\ Ch.\ 3.\ However,\ the\ expression\ is\ not\ positive\ definite\ function\ of\ \vec{x},\ e.g.\ see\ [9],\ Supplement\ II.\ Therefore,\ it\ does\ not\ suit\ as\ a\ probability\ density,\ although\ \int \rho(\vec{x}, t) d^3 x\ is\ positive\ and\ may\ be\ normalized\ to\ unity.
\]

Here I use Newton-Wigner wave function \( \Psi_{NW} \), see [10], Eq. (5). In their representation \( \rho(\vec{x}, t) = \Psi_{NW}^*(\vec{x}, t) \Psi_{NW}(\vec{x}, t) \), see [10], Eq. (6). In this equation and in what follows the letter \( \vec{x} \) denotes the Newton-Wigner coordinate, see [8], Ch. 3. The function \( \Psi_{NW} \) will be denoted by \( \Psi \).

The solution of (5) then may be represented as

\[
\Psi(\vec{x}, t) = (2\pi)^{-3/2} \int d^3 k \exp(i\vec{k}\vec{x}) \Phi(\vec{k}) \exp(-itE_k), \quad E_k = \sqrt{k^2 + m^2},
\] (6)

see [8], Chs. 7 and 3; [10], Eq. (5). \( \Phi(\vec{k}) \) is the initial wave function of the packet in momentum representation. For \( \Phi(\vec{k}) \) let us choose the product of three Gaussian packets

\[
\Phi(\vec{k}) = \varphi_1(k_1)\varphi_2(k_2)\varphi_3(k_3), \quad \varphi_j(k_j) = M \exp[-(k_j - p_j)^2\sigma^2].
\] (7)

The functions \( \varphi_j, j = 1, 2, 3, \) are normalized to unity

\[
\int dk_j |\varphi_j(k_j)|^2 = 1
\] (8)

if \( M^2 = \sigma \sqrt{2/\pi} \). Then \( \Phi(\vec{k}) \) is also normalized: \( \int d^3 k |\Phi(\vec{k})|^2 = 1 \).

It is easy to show that the parameters \( p_j \) in Eq. (7) are components of the mean momentum of the packet:

\[
\int d^3 k k_j |\Phi(\vec{k})|^2 = M^2 \int_{-\infty}^{+\infty} dk_j k_j |\varphi_j(k_j)|^2
\]

\[
= M^2 \int dk'_j (p_j + k'_j) \exp[-2(k'_j)^2\sigma^2] = p_j.
\] (9)
Packet Spreading and Einstein Retardation

The derivation uses the normalization (8), the change of the integration variables \( k'_j = k_j - p_j \), the parity of the function \( \exp[-2(k')^2\sigma^2] \).

The initial wave function in the coordinate representation \( \Psi(\vec{x}, 0) \), see Eq. (6) at \( t = 0 \), also reduces to the product of three factors

\[ \Psi(\vec{x}, 0) = \psi(x_1)\psi(x_2)\psi(x_3). \]

However \( \Psi(\vec{x}, t) \), Eq. (5), cannot be represented in such a simple form because of the factor

\[ \exp[-it(k_1^2 + k_2^2 + k_3^2 + m^2)^{1/2}] \]

in the integrand of Eq. (6).

The triple integral in Eq. (6) may be calculated approximately if the parameter \( \sigma \) is large enough, e.g. cf. [11], Ch. 3. To show this, let us change the integration variables \( \vec{k}' = \vec{p} - \vec{k} \) in Eq. (6):

\[ \Psi(\vec{x}, t) = (2\pi)^{-3/2}M^3 \int d^3k' \exp[i(\vec{p} - \vec{k}')\vec{x}] \times \exp[-(\vec{k}')^2\sigma^2] \exp\{-it[(\vec{p} - \vec{k}')^2 + m^2]^{1/2}\}. \]

The function \( \exp[-(\vec{k}')^2\sigma^2] \) cuts off the values of \( (\vec{k}')^2 \) which are much larger than \( 1/\sigma^2 \). So one may assume, e.g., \( k' < 3/\sigma \). Let \( \sigma \) be much larger than the Compton wave length \( \lambda_m = 1/m \), e.g. \( \sigma > 3\lambda_m \) or \( 3/\sigma < m \). It will be shown below that \( \sigma^2 \) is space dispersion of the initial packet (see Eq. (24)). It follows from the inequalities \( k' < 3/\sigma \) and \( 3/\sigma < m \) that \( k' \ll m \). Then \( k' \ll \sqrt{p^2 + m^2} \equiv E \) all the more. As \( k'/E \ll 1 \), one may expand

\[ \sqrt{(\vec{p} - \vec{k}')^2 + m^2} \approx \sqrt{p^2 + m^2 + (\vec{k}')^2 - 2(\vec{p}\vec{k}')} = E\sqrt{1 - 2(\vec{p}\vec{k}')/E^2 + (\vec{k}')^2/E^2} \]

in the series over degrees of \( k'/E \). Using the expansion

\[ \sqrt{1 + \alpha} = 1 + \alpha/2 - \alpha^2/8 + \ldots, \quad \alpha = -2(\vec{p}\vec{k}')/E^2 + (\vec{k}')^2/E^2 \]

and neglecting the term smaller than \( (k'/E)^2 \) one gets

\[ \sqrt{(\vec{p} - \vec{k}')^2 + m^2} \approx E[1 - (\vec{p}\vec{k}')/E^2 + (\vec{k}')^2/2E^2 - (\vec{v}\vec{k}')/2E^2], \quad \vec{v} = \vec{p}/E. \]

\( (11) \)
Let us direct the third axis \( \vec{e}_3 \) (\( \vec{e}_z \)) of the coordinate frame along \( \vec{p} \) so that \( \vec{p} = (0,0,p) \) and \( \vec{v} = (0,0,v) \) (\( p \) denotes \( |\vec{p}| \) and \( v \) denotes \( |\vec{v}| \)). In this frame (11) turns into

\[
\sqrt{(\vec{p} - \vec{k}')^2 + m^2} \cong E \{1 - pk'_2/E^2 + [(k'_1)^2 + (k'_2)^2 + (1 - v^2)(k'_3)^2]/2E^2\}. \tag{12}
\]

Note that no supposition on \( p \) value has been assumed so that \( 0 \leq p < \infty \). Using the approximation (12) in Eq. (10) one gets that the triple integral in Eq. (10) reduces to the product of three single-valued integrals:

\[
\Psi(\vec{x},t) \cong \psi_1(x_1,t)\psi_2(x_2,t)\psi_3(x_3,t),
\]

\[
\psi_1(x_1,t) = (2\pi)^{-1/2}M I_1(x_1,t); \quad \psi_2(x_2,t) = (2\pi)^{-1/2}M I_2(x_2,t); \tag{13}
\]

\[
\psi_3(x_3,t) = (2\pi)^{-1/2}M \exp[ipx_3 - iT]I_3(x_3,t); \tag{14}
\]

\[
I_j(x_j,t) = \int_{-\infty}^{+\infty} dk'_j \exp[-ik'_j(x_j - v_jt)] \exp[-i(k'_j)^2(a^2_j)], \tag{15}
\]

\[
a_1^2 = a_2^2 = \sigma^2 + it/2E, \quad a_3^2 = \sigma^2 + it(1 - v^2)/2E, \tag{16}
\]

\[
v_1 = v_2 = 0, \quad v_3 = v.
\]

For integrals \( I_j(x_j,t) \), Eq. (15), see e.g. [12], Ch. 2.5.36.1:

\[
I_j(x_j,t) = \sqrt{\pi/a_j} \exp[-(x_j - v_jt)^2/4a_j^2]. \tag{17}
\]

Using other tabular integrals one may verify that \( \psi_j(x_j,t) \), Eqs. (13), (14), (15), are normalized:

\[
\int dx_j |\psi_j(x_j,t)|^2 = (2\pi)^{-1}M^2 \int_{-\infty}^{+\infty} dx_j |I_j(x_j,t)|^2 = 1. \tag{18}
\]

Let us calculate the mean positions \( X_n(t) \), \( n = 1, 2, 3 \), of the moving packet at the moment \( t \):

\[
X_n(t) = \int \int \int d^3x_n |\Psi(\vec{x},t)|^2 \cong \int dx_n x_n |\psi_n(x_n,t)|^2
\]

\[
= (2\pi)^{-1}M^2 \int dx_n x_n |I_n(x_n,t)|^2. \tag{19}
\]
Here I use the normalization (18) of the function $\psi_j$ with $j \neq n$ and then use Eqs. (13), (14). Further the change $x_n' = x_n - v_n t$ of the integration variables is used in the last integral in Eq. (19). Finally, the parity of $I_n$ is taken into account: $I_n(x_n', t) = I_n(-x_n', t)$. One obtains the result

$$X_1(t) \cong X_2(t) \cong 0, \quad X_3 \cong vt, \quad v = |\vec{v}| = v_3 = p/E, \quad E = \sqrt{p^2 + m^2}. \quad (20)$$

This means that the center of the packet moves along $\vec{p}$ with the velocity $\vec{v} = \vec{\rho}/E$. In addition, the packet spreads, the spreading being characterized by the packet dispersions $\sigma_1^2, \sigma_2^2, \sigma_3^2$, see Eq. (4).

Let us name $\sigma_3^2(\vec{p}, t)$ the longitudinal dispersion $\sigma_3^2(\vec{p}, t)$ (dispersion along $\vec{p}$) and $\sigma_1^2, \sigma_2^2$ transversal ones. Using Eqs. (4), (19), (20) one obtains for $\sigma_3^2$:

$$\sigma_3^2(\vec{p}, t) \equiv \sigma_3^2(\vec{p}, t) = \int dx_3 x_3^2 |\psi_3(x_3, t)|^2 - (vt)^2$$

$$= (2\pi)^{-1} M^2 \int dx_3 x_3^2 |I_3(x_3, t)|^2 - (vt)^2. \quad (21)$$

The further derivation of $\sigma_1^2$ is more tedious than the calculation of $X_3(t)$. The result is

$$\sigma_1^2(\vec{p}, t) = \sigma^2 + t^2 (1 - v^2)^2 / 4E^2 \sigma^2, \quad E = \sqrt{p^2 + m^2} = m\gamma. \quad (22)$$

In the same manner one obtains the transversal dispersions

$$\sigma_1^2(\vec{p}, t) = \sigma_2^2(\vec{p}, t) = \sigma^2 + t^2 / 4E^2 \sigma^2. \quad (23)$$

If the packet is at rest ($v = 0, \ p = 0, \ E = m$) all dispersions are equal:

$$\sigma_1^2(0, t) = \sigma_2^2(0, t) = \sigma_3^2(0, t) = \sigma^2 + t^2 / 4m^2 \sigma^2. \quad (24)$$

When $t = 0$ Eqs. (21), (22) turn into the initial dispersions:

$$\sigma_1^2(\vec{p}, 0) = \sigma_2^2(\vec{p}, 0) = \sigma_3^2(\vec{p}, 0) = \sigma^2.$$

This equation makes clear the physical meaning of the parameter $\sigma$ in Eq. (7).
Note. The quantities $\sigma_j^2(t)$ occurred in paper [13], see Eq. (24) in App. A. There they played the role of notations, their physical meaning being not revealed. It was shown here that these quantities (denoted as $\sigma_j^2(\vec{p}, t)$ here) do have the meaning of packet dispersions at the moment $t$. Remark also the error in writing the expression for $\sigma_3^2(t)$ in Eq. (24) in [11]: there $(1 - v^2)$ must be squared, cf. Eq. (22) here.

3 Discussion

Let us compare the dispersions of the moving packet and the packet at rest. Using the designations

$$E = \sqrt{p^2 + m^2} = \gamma m, \quad \gamma = (1 - v^2)^{-1/2},$$

rewrite Eqs. (21), (22) in the form

$$\sigma_1^2(\vec{p}, t) = \sigma^2 + t^2/\gamma^6 m^2 \sigma^2, \quad (25)$$
$$\sigma_2^2(\vec{p}, t) = \sigma_2^2(\vec{p}, t) = \sigma^2 + t^2/\gamma^2 m^2 \sigma^2. \quad (26)$$

Comparing with the dispersions $\sigma^2(0, t)$ of the packet at rest, see Eq. (24), one obtains

$$\sigma_1^2(\vec{p}, t) = \sigma^2(0, t/\gamma^3), \quad (27)$$
$$\sigma_2^2(\vec{p}, t) = \sigma_2^2(\vec{p}, t) = \sigma^2(0, t/\gamma). \quad (28)$$

The transversal dispersions of the moving packet evolves slower than the dispersions of the packet at rest. Its slowing down is Einsteinian: the dispersions $\sigma_1^2(\vec{p}, t)$ and $\sigma_2^2(\vec{p}, t)$ at the moment $t$ assume the value which $\sigma^2(0, t)$ assumes at the earlier moment $1/\gamma$ (see ER definition in the Introduction).

The longitudinal spreading $\sigma_3^2$ also grows slower than $\sigma^2(0, t)$, but the retardation of $\sigma_3^2$ is not ER, see Eq. (27). So ER fails. I suppose that this result deserves its detailed derivation in Sect. 2. The derivation used the following premises.

The packet of scalar (spinless) particle is described by the wave function $\Psi(\vec{x}, t)$ which satisfies relativistic positive-energy Klein-Gordon equation.
Packet Spreading and Einstein Retardation

The initial packet state is described by the simple Gaussian function of macroscopical space size.

To calculate $\Psi(\vec{x}, t)$, the usual approximation was exploited, see Eq. (12).

Using $\Psi(\vec{x}, t)$ the packet dispersions at the moment $t$ were obtained. Unlike $\Psi(\vec{x}, t)$ the dispersions are the observable nonstationary quantities which can be experimentally measured.

Examples of nonstationary systems for which ER fails were given in [5]-[7]. The considered system complements the examples. So one may state that ER is not a universal (kinematic) law for quantum clocks.

However, in the case of unstable particles quantum mechanics shows that ER holds with high precision [3]-[5]. Experiments also agree with ER, see e.g. the corresponding references in [3]-[6]. As was argued in Introduction, usual explanation of ER (based on Lorentzian transformations of position and time) is nonapplicable for quantum clocks. It is quantum mechanics which provides the suitable theoretical explanation.

References


Shirokov considers the dispersion $\sigma(0, t)$ of a positive-energy wave function for a scalar relativistic particle with zero mean momentum. In a moving frame, $v = p/m$, the dispersion is denoted by $\sigma(p, t)$ . Shirokov exhibits an example where $p = pe_3$ and where the dispersion, $\sigma_3$, for the $x_3$ direction is given by

$$\sigma_3(p, t) = \sigma_3(0, t/\gamma^3), \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. $$

He argues that the dispersion may be regarded as a clock and that one should therefore expect an Einsteinian retardation $\gamma/t$ instead of $\gamma/t^3$. 
The following observations immediately come to mind.

- The determination of the dispersion requires measurements over all space. Moreover, for large times, the spreading increases to infinity so that the "clock" is spread over all space. In contrast, classical clocks are assumed to be localized, and the usual statements of relativity theory depend on this.

- Shirokov uses the Newton-Wigner position operator to calculate $\sigma$. However, it is known that this operator possesses unphysical causality properties [1]. Moreover, these unphysical properties are shared by any conceivable choice of a self-adjoint position operator [2]. Is the $t/\gamma^3$ result also true for more general operators of localization?

- The approximation used in Eqs. (11/12) seems to imply a restriction on $t$ because the approximation is performed in a phase and therefore one must have that the error multiplied by $t$ is much less than.

Interesting as the behavior of the dispersion for moving particles may be - even when employing the not very satisfactory Newton-Wigner position operator - in my opinion its interpretation in the context of quantum and relativity theory warrants further discussion.

References


DISCUSSION AND REPLIES

In this Section we publish comments, replies and discussions on previously published papers. We encourage the Readers to express their critical opinions.

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Author of [1] addresses one of the key questions in cosmology, interpretation of the cosmological redshift (Hubble law). The standard cosmology, the Friedman model, gives interpretation of the cosmological redshift in terms of the expanding universe. Recall that the Friedman model describes the evolution of the scale factor of the universe with time, i.e., stretching of the space of the universe with time. The question arises as to what the universe expands. Quoting reference in [1], ”but why is then, say, Brooklyn, not expanding?” In the Friedmann model the universe expands at the large scale relative to the gravitationally bound scale. Then, in the Friedmann model, the Hubble law is valid for distances which are larger than the size of the cell of homogeneity, 100-150 Mpc or more. But observations [2] tell us that the Hubble law is seen not only at large distances but at distances from 1.5-2 to 20 Mpc as well. Putting aside confrontation of the theory against observation, the Friedmann model regards in fact two universes, static for bound structures and expanding for the whole universe. One can see different bound structures in the universe from the electron scale \(10^{-13}\) cm to the supercluster scale 100 Mpc. These structures behave such if there is no expansion. The question arises how the two universes, static and expanding, may coexist. Yet, it is reasonable to have a unique universe which follows universal laws of scale invariance.

Instead of the idea of the stretching space, another interpretation is discussed elsewhere, e.g. [3], that locally recession velocities associated with the cosmological redshift are kinematic. Author of [1] also follows this idea. The interpretation signifies that we proceed from the Friedmann model to the Milne model wherein the particles fly away in the empty space. Then, the cosmological redshift is treated within special relativity as a Doppler effect. Nonetheless, the question relative to what the universe expands (particles move) remains unanswered. A particle in the Milne model, on the one hand, belongs to some bound (static) structure, on the other hand, is involved in the cosmological motion. In the Milne model we also have two universes, static and expanding, hence the problem of coexistence of the two universes remains unresolved.

Interpretation of the cosmological redshift in the static universe is proposed in [4]. Consider the static universe in the background Euclidean space and the absolute time. Let the universe have a finite
size \( R \). Suppose that the rest frame at the distance \( r \) is equivalent to the frame receding with the velocity

\[
v = \frac{cr}{R}
\]

where \( c \) is the speed of light. Write down the relativistic shift for the wavelength of light between two frames. The wavelength of light coming from the distance \( r \) is shifted in the frame of receiver as

\[
\lambda_r = \lambda(1 + v/c) = \lambda(1 + r/R).
\]

This shift of the wavelength of light may be interpreted as the Hubble law

\[
z \equiv \frac{\lambda_r - \lambda}{\lambda} = \frac{r}{R} = \frac{Hr}{c}
\]

where \( H = c/R \) is the Hubble parameter. Thus, the shift for the wavelength of light between two rest frames at the distance \( r \) is the same as that between two moving frames with the velocity of recession given by eq. (1).

One can interpret the redshift depending on distance in terms of the relative velocity between two frames that seems to contradict to the static universe. But one should consider the effect relative to the whole universe. Since the velocity is relative one can take the frame at the distance \( r \) as a rest frame and the laboratory frame as a moving frame. Then, an observer in the laboratory frame determines the velocity \(-v\) with respect to the rest frame at the distance \( r \) and the velocity \( v \) with respect to the opposite rest frame at the distance \(-r\). The total velocity of the laboratory frame with respect to the both rest frames at the distance \( r \) is equal to zero. Hence, the laboratory frame has the null velocity with respect to the whole (static) universe.

In conclusion, the standard cosmology has no reasonable answer to the question relative to what the universe expands. Interpretation considered by author of [1] which treats the cosmology in terms of moving particles in the empty space (Milne model) instead of the space stretching with time (Friedman model) also gives no plausible answer to the question. A possible resolution of the problem is to consider the relativistic effect depending on distance [4] which explains cosmological redshift in the static universe. The relativistic
shift depending on distance mathematically explains the cosmological redshift in a similar way as a conventional Doppler effect however the interpretation is different. We come to the static universe whereas the expansion of the universe due to the cosmological redshift turns out to be mere an illusion.

References


Khokhlov raises an important question, namely “relative to what the Universe expands”. Indeed, even in Newton’s theory there is Galilean relativity, and absolute motion does not exist. In the paper under discussion, my answer to this question is provided by, perhaps in a too ‘cryptic’ way, in the last-but-one sentence: “all that matters is the cosmic substratum (fluid) and its relative motions”. In Chodorowski (2007), I further elaborated in somewhat more detail that the cosmological equations “describe nothing more than real relative motions of the particles of the cosmic substratum”. In both cases, I meant that it is the cosmic substratum that is really expanding, not the Universe ‘in its own right’. One cannot decouple the expansion of the Universe from the matter that is participating in the expansion. We can sensibly define, and detect through observations, only the motions of the particles of the cosmic substratum.
However, Khokhlov seems to be unsatisfied even with such an interpretation of the Universe’s expansion. He writes: “the question relative to what (…) particles move remains unanswered”. I disagree. The particles move, of course, relative to each other. Here, the analogy with an inflated balloon with dots (representing galaxies) drawn on its surface is to some extent useful. During inflation of the balloon distances between all dots increase. An important limitation of this picture is that “in reality, the balloon does not exist; there are only dots” (Chodorowski 2008). And they really move, in accordance with the generally-relativistic, cosmological equations.

Khokhlov proposes an alternative mechanism for the generation of the cosmological redshift. As the background cosmological model he adopts a static universe. The static model is not a viable cosmological model, but, just like in my paper under discussion, can be used as a toy model to understand physics. This is so because the spacetime of a very low density universe is approximately given by Minkowski spacetime, which is static.

Unfortunately, Khokhlov’s mechanism for the generation of the cosmological redshift in a static universe is wrong. (The way it is presented is also unclear.) While in the standard approach, the particles of the cosmic substratum move according to the Hubble flow, in his approach they seem to be at rest relative to each other. In the sentence ‘suppose that the rest frame at the distance $r$ is equivalent to the rest frame receding with the velocity $v...’$, what does ‘equivalent’ exactly mean? Just that it produces the same redshift? In any case, the assumed $v$ is numerically equal to the recession velocity of a galaxy located at a distance $r$, partaking in the Hubble flow. In eq. (2) Khokhlov uses eq. (1) for the velocity and resulting eq. (3) is in fact nothing other than a formula for the Doppler shift of the receding galaxy (by the way, correct only for non-relativistic velocities). Therefore, the Author’s conclusion that ‘the shift for the wavelength of light between two rest frames at the distance $r$ is the same as that between two moving frames with the velocity of recession given by eq. (1)’ is in fact a tautology. And how, under the assumption of no gravity, which has been adopted here, a separation between two frames can alone generate redshift? A photon, travelling from the source’s frame to the observer’s frame, where both frames are at rest relative to each other, undergoes a redshift proportional to the cov-
Answer to the comment by Khokhlov

... ered distance?? This is really ‘new physics’, in conflict with everyday experience and countless physical experiments.

Modifying old theories, or, even more, abolishing them and inventing new ones is the most ambitious and fascinating part of the scientific research. However, if you want to criticize an existing theory rationally you should know it well. I am convinced that Dr. Khokhlov will find more easily correct answers to his interesting questions if he acquires his theoretical knowledge of cosmology not from the papers by Sandage, but from the modern textbooks.

References


The aim of this Section is to publish articles which recall interesting episodes from the history of physics. We welcome articles reminding “forgotten” papers, facts, controversies and disputes.
DESCARTES’ VACUUM IN HOLE
GRAVITATION THEORY

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Abstract

Intuitively, a vacuum is what is left when all matter (invisible as well as visible) is removed from a region. In this paper it is shown that depending on the speed we remove the matter, we can obtain a Torricelli’ vacuum or a Descartes’ absolute vacuum (a hole in space-time). The advantage of the Descartes’ vacuum is that it can easily explain the gravitational attraction and the curvature of space-time using the properties of holes only. According to the theory, matter radiates a flux of holes, which curve space-time and this curvature influences the movement of matter. The hole version of universal gravitation law has been formulated. Mass is the parameter describing the ability of a particle to emit out holes, the more holes particle emits, the more the mass.
1 Introduction

Understanding the nature of mass is crucial in fundamental research. Since attempts to detect the Higgs boson, and therefore to verify the Higgs field as the mass-generating mechanism of the Standard Model, have been unsuccessful, paper proposes a very simple model of gravitation that use the Descartes’ void (holes in space-time). The advantage of the present approach is that it can easily explain the curvature of space-time using the properties of Descartes’ vacuum only. Gravitation is a phenomenon by which all objects with mass attract each other. Hole theory of gravitation explains why object have mass, how all objects with mass attract each other. Matter interacts with space-time that and radiates a flux of holes that is the cause of the gravitation. Thus for explanation of gravitation the present theory use the hole structure of space-time only.

2 Holes in space-time

Descartes maintained that there could be no vacuum, and all matter was constantly swirling to prevent a void as corpuscles moved through other matter. Nevertheless, Descartes first describes the new type of absolute void. Let’s analyze the main ideas in Descartes theory [1]: “if it be asked what would happen were God to remove from a vessel all the body contained in it, without permitting another body to occupy its place, the answer must be that the sides of the vessel would thus come into proximity with each other. For two bodies must touch each other when there is nothing between them, and it is manifestly contradictory for two bodies to be apart, in other words, that there should be a distance between them, and this distance yet be nothing; for all distance is a mode of extension, and cannot therefore exist without an extended substance”. Thus the main positions of Descartes theory [2] are: 1. If to remove from a vessel all the body contained in it, without permitting another body to occupy its place, the sides of the vessel would thus come into proximity with each other; 2. All matter was constantly swirling to prevent a void; 3. it being absolutely contradictory that “nothing” should possess extension. It means that the absolute Descartes vacuum do not have the property of extension. It is important to notice that in modern physics the concept of space-time combines space and time within
Descartes' vacuum in Hole Gravitation theory

a single coordinate system. In the Theory of Relativity, time is no more an independent physical quantity, it is linked with space in four-dimensional space-time. Consequently, inside of Descartes vessel is no extension and no time too, it is a hole in space-time. In such a way a hole in metal, for example, do not have the properties of metal. Probable, Descartes does not mention about time in his vessel because in past physical time was an independent quantity (absolute time), running uniformly throughout the entire cosmic. Let we repeat the Descartes experiment with vacuum. There is a vessel contained a gas (body). Consider a gas has been completely evacuated by pumps and other devises, so that the (classical) particle concentration is zero. According to Descartes, the sides of the vessel would thus come into proximity with each other. Nevertheless, we see the vessel intact, in spite of the fact that a vessel contains a vacuum. According to Descartes, “nothing cannot possess extension”; consequently, our vessel is not empty and contains invisible particles. It means another “body” (or particles) occupied the vessel’s space during a time we evacuate a gas. It explains why the walls of the vessel do not come into proximity with each other. What particles fill our “empty” vessel? These particles are able to penetrate through any material walls, and could not be removed neither by the pumps, nor by other similar devices. Particles are invisible, but possessing the property of extension. Probably it is neutrino, fundamental fields, radiation or virtual pairs particle-antiparticle which are present in the vacuum. Besides some theories affirm that space is quantized and consist of elementary volumes dV or space cells. The space cells have just the property of extension, are invisible and cannot be removed from vessel neither by pumps nor by other similar devises. Thus, the conclusion is that during time we evacuate a gas from vessel, the free space occupies another particles. But even so, the main question remains unresolved about if there could be the Descartes’ void in our chamber. How can we create a Descartes’ vacuum? We must remove from a vessel all the body contained in it, without permitting another body to occupy its place.

There is a solution to use Einstein’s relativity to produce holes; no signal can be transmitted faster than c. Since the speed of motion is limited by the speed of light c, if we remove the body from vessel very quickly (instantly), the sides of the vessel would thus come
into proximity with each other, because environment cannot occupy the vessel’s space instantly. Because the walls of vessel are material and cannot move faster than light in order to come into proximity instantly. Therefore the lifetime of Descartes void or a hole in space-time is nonzero.

There are two kinds of the physical vacuum: a Torricelli’ vacuum and a Descartes’ vacuum. First we obtain if the body is removed slowly from the vessel due to the space of vessel occupies quickly other particles; Second absolute void we obtain if we remove the body instantly. In this case the Descartes void exist during a short time while walls of the vessel come into proximity with each other at near-light speeds. In other words, a Torricelli’ vacuum is a volume of space that is essentially empty of matter, such that its gaseous pressure is much less than atmospheric pressure. In contrast, a Descartes’ vacuum is a total absence of both matter and space-time (in vessel) due to the walls of vessel come into proximity with each other. The Descartes vacuum (a hole in space-time) can be directly deduced also from the theory of quantized vacuum. Lets analyze the implications of quantized space-time - space being granular, not continuous, at its smallest scales. Suppose that space-time is composed of a fluctuating space cells or space atoms (elementary volumes dV). Such particles are invisible and Lorenz invariant because they are virtual particles that appear and disappear continuously. If the space cell disappears (instantly), in the same “place” appears a vacant place or a hole in space-time. Then particles of surrounding medium (space cells and elementary particles) fill a hole as a Descartes’ vessel. In that case the virtual holes and “Descartes’ vessels” must fill the entire Universe, if the space-time has quantum structure. It is enough to construct the present Hole Gravitation theory. If one space cell disappears with creation of hole, the surrounding particles fill the hole as show figure 1. According to Descartes, such holes must have the following properties:

1. Descartes theory affirms that a hole in space-time cannot have the extension properties, because “it being absolutely contradictory that nothing should possess extension” [1]. In addition, it is logically evident that a hole in space-time cannot have also the time properties, because it is a hole in space-time. More precisely, the properties of extension and time of hole tends to zero. Very close to a hole, time
Descartes' vacuum in Hole Gravitation theory

virtually stands still for the outside observer.

2. Therefore, we cannot observe a hole by definition, because “it is nothing”, but we can observe the Descartes vessel - the material walls of vessel that would thus come into proximity with each other;

3. The lifetime of the hole is nonzero because surrounding particles (the walls of vessel) cannot come into proximity with each other instantly (because the speed of motion is limited by the speed of light $c$). Therefore the lifetime of hole cannot be less than $R/c$. ($R$ is the radius of hole, $c$ - the speed of light).

3 Hole gravitation theory

Thus, space-time is quantized and consists of fluctuating space cells $dV$ which appear and disappear continuously. When space cell disappears, instead appears a hole in space-time. Therefore, the holes in space-time appear continuously in random points. We analyze interaction of a material particle $P$ with the vacuum holes.

![Figure 1: The interaction of particle $P$ with one (a) and two holes (b). It is a Descartes vessel with a hole in the centre (the white spheres), walls composed of material particle $P$ (black sphere), and space cells (grey spheres).](image)

Around particle $P$ continuously appear and disappear holes in space-time. Consider the interaction between a particle $P$ and a vacuum hole (Figure 1a). The appearance of the vacuum hole near $P$ means that a Descartes void has appeared which must be filled by all surrounding particles (by particle $P$ and surrounding space cells $dV$). In fact, it is a Descartes vessel, where white spheres are the vacuum holes and grey spheres are the walls of vessel. Therefore a particle $P$ and space cells $dV$ will move to the centre of hole. Let’s
consider now a case when two vacuum holes appear simultaneously on opposite sides of particle, as show figure 1b. The particle cannot fill simultaneously two holes and cannot move to the opposite sides; therefore particle will stand fixed while both vacuum holes will be filled by surrounding space cells (dV) only. Since the dV moves to particle, it means that the holes move to the opposite side. There is analogue with an electric current where electrons move in one direction and holes move in the opposite. As vacuum holes continuously appear around particle, it means that particle will emit continuously a flux of “their own” holes. It is necessary to notice, that dV moves to particle in both cases a and b (Figure 1). Therefore, particle radiates holes in both cases a and b. Thus, a massive material body must emit a flux of holes by each component particle. The speed of motion of vacuum hole in space should be equal to the “collapse” speed of hole that is supposed to be equal to the speed of light c. The definition of mass: The mass of a particle is a parameter describing the ability of a particle to interact with vacuum holes and emit “its own” holes; the more holes a particle radiates, the more the mass.

4 The hole version of universal gravitation law

If to collect all holes emitted by a material point during a time unit (one second), we shall receive a sphere with volume $V$ and radius $r$. In that case the definition of universal gravitation can be formulated in the following manner: There is a force of mutual attraction between two points directly proportional to cubes of radiuses of summarized volumes of holes emitted in time unit and inversely proportional to square of distance between them. If the two bodies emits during a second a volume of holes $V_1$ and $V_2$, with radiuses $r_1$ and $r_2$, and the distance between them is $R$, the magnitude of the force is:

$$F = G_m \frac{r_1^3 r_2^3}{R^2} \tag{1}$$

Where $G_m$ is a metric gravitational constant equal to $Gm = 1.665 \times 10^9 N/m^4$ (or $kg/m^3 s^2$); R is the distance between two points that emits holes (or between the centers of mass of interacting bodies); If bodies are small compared with the distance R, or if they are spherical, expression (1) is correct as it stands; for non-spherical
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shapes the acceleration has to be calculated separately for each part of the bodies and then added together. The centre of summarized volume of holes $V$ obviously coincides with the center of mass of body. From formula (1) follows that $G_m$ (in Newtons) is a force of attraction between two points that emits during a second a stream of holes with summarized volume $V_0$, which is a sphere with radius one meter, and the distance between points are $R = 1$ m. Also the formula 1 works only if a hole field is weak and bodies move slowly in comparison with the speed of light. Formula 1 use pointlike sources of holes whereas the real picture is some different. A material body radiates holes by each component particle due to it creates some distribution of holes in space (the curvature of space-time). Because of these “simplifications” in above formula appear errors as discrepancy in Mercury’s orbit. It is known that orbits precess in a way unexpected in Newton’s theory of universal gravitation law. The main parameter of hole gravitation as the summarized volume $V_0$ can be calculated without using of notion “mass” or Newton gravitational law, proceeding from geometrical reasons only. If we have measured the acceleration of free fall $g$ of test body, the summarized volume of holes $V_0$ emitted during a second by gravitating body is:

$$V_0 = \frac{4\pi(R^3 - (R - g)^3)}{3}$$  (2)

Hole theory of gravitation could, therefore, be strictly considered as independent of the concept of mass, and the fact that formula (1) can be transformed very simply to Newtons formula is the proof of its validity only. Actually, why is the concept of mass necessary in general? The concept of “mass” was introduced in antiquities for such purposes as trade, construction etc. But now this physical parameter simply duplicates such fundamental concepts as length, volume, and time. It is possible to exclude completely the concept of mass by measuring the inertia and gravitation of body in volume units $m^3$ the volume of holes emitted by body in time unit. There 1 kg is equivalent to the volume $4\pi G_v$ (cubic meters). The volume $V_0$ relates to mass by expression: $V_0 = 4\pi G_v M$; where $M$ is the mass of body, $G_v = 6.672 \times 10^{-11} \ m^3/kg$ the coefficient of transformation of mass in volume, that numerically is equal to gravitational constant, but with other units of measurements $m^3/kg$. The summarized volume

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of holes emitted by a body that exist at the distance $r >> R$ from the centre of hole $V_0$ can be represented as

$$g = \frac{r^3}{3R^2}$$  \hspace{1cm} (3)

There $g$ can be understood as the acceleration of free fall. We can explain now the curvature of space-time using properties of Descartes vacuum (holes in space-time) only. The properties of space, as well as properties of any body should depend on its component particles. For example, if to increase the concentration of holes in space, the properties of space should be displaced to properties of the hole. The main property of a hole (Descartes vacuum) is that the extension property tends to zero and time runs infinitely slow. Therefore, if we increase the concentration of holes in space, it would results in contraction of all distances between any two points and time retardation, because in the limiting case when space consists of holes only, the distance between any two points are equal to zero and time runs infinitely slow. The given effect of length contraction and time retardation near massive bodies was called a curved space-time. Hole gravitation describes gravitation as the influence of physical matter on properties of space-time which in turn influences the movement of a matter and other physical processes: The matter bends space-time by emission of streams of holes, and this curvature shown as gravitation, influences the movement of matter.

5 The geometrical explanation of gravitational attraction using holes

We may show the gravitational attraction between two bodies by means of the holes, using the property of holes to collapse as a Descartes vessel. It is difficult to show the attraction process with every elementary hole, therefore we shall collect all holes emitted by mass $M$ of the Earth during a second, having received a hole with volume $V_0 = 4\pi G_v M$. Thus, it is possible to substitute the Earth by equivalent hole $V_0$ which collapses every second.

Let the test body $N$ exist at rest on distance $R$ from the centre of hole $V_0$, as show Figure 2. At the moment of time $t_0$, a hole $V_0$ begins to collapse and to the moment $t_1$ the volume $V_0$ will be equal
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Figure 2: The test body $N$ exists at rest on distance $R$ from the hole $V_0$.

to zero. Therefore, the sphere $V$ decreases on size: $V - V_0 = V_1$ due to a hole with diameter $g$ appears between $N$ and sphere $V$.

$$g = R - R_1 = R - \sqrt[3]{R^3 - r_0^3} = 9.8m$$

(4)

Now a hole $g$ collapses as a Descartes vessel, where body $N$ and Earth are the walls of vessel. Therefore both $N$ and Earth will move with acceleration to the centre of hole $g$, and $N$ passes the distance $S_1 = 4.9m$ up to the centre of hole, but with the speed $V = 9.8m/s$ concerning the Earth. The next second $t_2$ body $N$ passes the distance of $9.8m$ by inertia, and besides, is again accelerated by a hole $g$, moving to the centre of hole $g$ together with Earth the distance $4.9m$. Thus the body has passed distance $S = S_1 + 9.8 + 4.9 = 19.6m$. All things considered, an object starting from rest will attain a speed of $9.8m/s$ after one second, $19.6m/s$ after two seconds, and so on.

Continuing this experiment we shall find, that the body moves by the law

$$S = \frac{gt^2}{2}$$

(5)

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which describes the free falling in a gravitational field. Thus the material bodies fall in a hole field just as in a gravitational field, consequently hole radiation is a gravitation. In the hole theory, the effects of gravitation are ascribed to space-time curvature instead of a force. The cause of gravitation is due to vacuum holes change the geometry of space-time, which causes inertially moving objects (Earth and test body) to tend to accelerate towards each other (without any forces). The hole theory explains both Newton and Einstein gravitation theories and makes following predictions:

1. Hole theory predicts the gravitational contraction and time dilation near the source of vacuum holes (a massive body), because in the Descartes vacuum the property of extension tends to zero and time dilation is infinite.

2. Hole theory predicts that the speed of gravity must be equal to the speed of light in vacuum because a hole cannot collapse faster than light. Therefore, the speed of gravitation cannot be faster than light.

3. Hole theory predicts the fluctuation of space-time geometry at subatomic scales because of appearance of holes in space-time. Since “inside of hole” the property of extension tends to zero and time dilation is infinite, the appearance of holes led to the fluctuation of geometry.

4. It explains the absence of the gravitational repulsion - the hole model allow attraction only.

5. The theory explains why no graviton has ever been detected; because a vacuum hole is “nothing” and cannot be observed or detected by definition. We cannot see or detect “nothing”, but we can find the source of holes indirectly via detection of the gravitational contraction and time dilation.

6 Conclusions

In this paper I have tried to explore the Descartes ideas about vacuum in order to build the gravitation theory. Gravitation appears simply because of interaction between matter and space-time, causing the holes to flow. It is then easy to see that the length contraction and time dilation near massive body appear because of properties of Descartes’ vacuum. Also the hole theory explains gravitational attraction using geometrical reasons only.
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References


ESSAYS
Quanti\textup{um} Theory Without Logical Paradoxes?

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Abstract

In the contribution published in AIP Conference Proceedings, No. 1018 (p. 40-5) some discrepancies in Copenhagen quantum mechanics have been summarized and arguments have been gathered why in the description of microscopic reality hidden-variable theory should be preferred. It has been shown, too, that the earlier gap in physical descriptions of microscopic and macroscopic worlds has been practically removed. All corresponding statements have been based on proofs that had been published earlier and reproduced in the given contribution only partially. In the present paper the whole approach will be explained more systematically. Some important points newly analyzed will be added.
1 Introduction

In our preceding papers (see mainly [1, 2]) we listed the arguments which led to the conclusion that the hidden-variable (HV) theory should be preferred to the Copenhagen quantum mechanics. The results based on these arguments have been presented to the conference held in Trieste (January 2008) and published in corresponding Proceedings [3] (similar results being contained also in Ref. [4]). In the present paper the more systematic explanation of the whole approach will be provided.

The new results may be shortly summarized as follows: Both Copenhagen and ensemble interpretations of quantum-mechanical model represent in fact two different theories with important difference in assumptions. The ensemble alternative being practically identical with the mere Schrödinger equation (and equivalent to hidden-variable theory) in principle refuses the physically interpretable superposition principle and requires the extension of the corresponding Hilbert space to be in full harmony with the time-dependent solutions of the Schrödinger equation[5]. All often discussed previous criticism (Pauli, Susskind and Glogover, Einstein) is removed in such a case. And no logical contradictions and paradoxes are anymore involved in the description of matter reality. It has been also proved that the basic solutions (represented always by one Hamiltonian eigenfunction) of Schrödinger equation lead to the same results as classical physics. However, some classically possible states do not seem to correspond to those basic Schrödinger solutions, which describe discrete states. It is also possible to say that the microscopic physics may provide now the same ontological picture as it is known from classical physics.

Main assumptions on which the Copenhagen quantum mechanics is based will be introduced in Sec. 2 and the difference between two earlier often discussed interpretational alternatives will be explained. Critical comments of Pauli [6] and others, discussed during the second half of the 20th century, will be explained in Sec. 3. The necessity of extended Hilbert space and time irreversibility will be then handled in Sec. 4. The story of the EPR experiment will be described and explained in Sec. 5. And in Sec. 6 the physical meanings of different limits of Bell’s operator will be shown. The possibility of applying the HV (hidden-variable) theory to the whole physical reality will be discussed in Sec. 7. The results and consequences of the experiment
with three polarizers will be presented in Sec. 8. The actual relation between the Schrödinger equation and classical physics will be then shown in Sec. 9. The structure of Hilbert space corresponding to the HV theory will be discussed in Sec. 10. Some concluding remarks will be given in Sec. 11.

2 Copenhagen quantum mechanics and basic assumptions

To understand the whole problem it is necessary to start with main assumptions on which the Copenhagen quantum mechanics is based. Leaving aside some technical assumptions it is possible to list four following ones:

- first of all, it is the validity of time-dependent Schrödinger equation \[ \left( \frac{i\hbar}{\hbar} \frac{\partial}{\partial t} \right) \psi(x, t) = H \psi(x, t), \quad H = -\frac{\hbar^2}{2m} \Delta + V(x), \] (1) where the Hamiltonian \( H \) represents the total (kinetic and potential) energy of a given physical system and \( x \) represents the coordinates of all matter objects;

- second, the evolution of a physical system is described by the function \( \psi(x, t) \) and all physical quantities may be expressed as the expected values of corresponding operators:

\[ A(t) = \int \psi^*(x, t) A_{op} \psi(x, t) dx, \] (2)

where \( A_{op} \) and functions \( \psi(x, t) \) may be represented by operators and vectors in a suitable Hilbert space;

- third, in the case of Copenhagen quantum mechanics it has been required that the corresponding Hilbert space is spanned on one set of Hamiltonian eigenfunctions \( \psi_E(x) \):

\[ H \psi_E(x) = E \psi_E(x), \] (3)

- and, finally, the mathematical superposition principle valid in any Hilbert space has been interpreted in physical sense, i.e., any
superposition of two vectors represents again another pure (basic) physical state.

It has been spoken often about two different interpretational alternatives of the quantum-mechanical mathematical model: the orthodox (or Copenhagen) one and the statistical (or ensemble) one. However, it has never been sufficiently stressed that both these alternatives correspond to two different sets of basic assumptions. While the Copenhagen alternative has involved all four preceding assumptions the ensemble alternative (denoted usually also as HV theory) correspond to the first two assumptions only (being equivalent in principle to the mere Schrödinger equation). And it is necessary to speak about two different theories, differing significantly in their properties as well as in assumptions.

The Copenhagen alternative has been often denoted as supported by different experimental data. However, in all such cases only the assumption of the mere Schrödinger equation (i.e., of the HV theory) has been tested; without the last two assumptions (forming Copenhagen alternative) having been actually involved. All four assumptions have been involved practically only in interpreting the EPR experiment, which will be discussed in Sec. 5.

As for the HV theory the Hilbert space must be chosen according to the corresponding physical system; it must be always extended (at least doubled) in comparison to the Hilbert space required by the third assumption. In such a suitably extended Hilbert space the critical comments of Pauli and also of Susskind and Glogover are no more valid (see the next section).

3 Critical comments of Pauli and others

The Copenhagen quantum mechanics was accepted as a valid theory by physical community even though it exhibited some logical and ontological paradoxes and some critical comments were brought against its full regularity. Already in 1933 Pauli [6] showed that under the validity of all assumptions introduced in Sec. 2 it is necessary that the corresponding Hamiltonian possesses continuous energy spectrum from $-\infty$ to $+\infty$, which disagrees with the fact that the energy is defined as positive quantity, or at least always limited from below. In 1964 Susskind and Glogover [7] showed that exponential
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The phase operator is not unitary, which indicated that the given Hilbert space is not fully complete to represent a corresponding physical system quite regularly. Many attempts have been done to solve these deficiencies during the 20th century. The reason of having been unsuccessful may be seen in the fact that practically in all cases both the shortages were regarded and being solved as one common problem.

The corresponding solution seems to have been formulated only recently (see Refs. [8, 9]) when it has been shown that it is necessary to remove two mentioned shortages one after the other. As to the simple system of two free colliding particles the criticism of Pauli may be removed if the Hilbert space required by the third assumption has been doubled as proposed by Lax and Phillips in 1967 (see [10, 11]); it consists then at least of two mutually orthogonal subspaces \( \mathcal{H} = \Delta^- \oplus \Delta^+ \); each of them being spanned on one basis of Hamiltonian eigenfunctions (see also Sec. 4).

As to the non-unitarity of the exponential phase operator it is necessary to mention that this operator was discussed for the first time by Dirac [12] in the case of linear harmonic oscillator (i.e., \( V(x) = kq^2 \)). Annihilation and creation operators

\[
a = p - im\omega q, \quad a^\dagger = p + im\omega q, \quad \omega = \sqrt{\frac{k}{m}}
\]

were introduced, fulfilling the relations

\[
[H, a] = -\omega a, \quad [H, a^\dagger] = \omega a^\dagger.
\] (5)

It was then possible to define an operator

\[
\mathcal{E} = (aa^\dagger + 1)^{1/2} a, \quad \mathcal{E}^\dagger = a^\dagger(aa^\dagger + 1)^{1/2}
\] (6)

fulfilling the relations

\[
[H, \mathcal{E}] = -\omega \mathcal{E}, \quad [H, \mathcal{E}^\dagger] = +\omega \mathcal{E}^\dagger
\] (7)

and representing exponential phase operator defined as

\[
\mathcal{E} = e^{-i\omega \Phi}
\] (8)

where \( \Phi \) is the phase.

And it was shown later by Susskind and Glogower [7] that in the Hilbert space corresponding to the third assumption the operator \( \mathcal{E} \)
was not unitary, but only isometric, as it held $\mathcal{E}^\dagger \mathcal{E} u_{1/2} \equiv 0$. The unitarity condition has not been fulfilled for the state vector corresponding to the minimum-energy (or vacuum) state. The Hilbert space (extended to solve Pauli’s problem) should be further doubled and formed by combining two mutually orthogonal subspaces corresponding to systems with opposite angular momentums. They should be bound together by the added action of exponential phase operator, linking together the corresponding vacuum states as it was proposed already by Fain [13]; see also [8, 9]. In any case, the given non-unitarity of exponential phase operator has represented always a less substantial problem than the criticism of Pauli, as it has concerned the completeness of the Hilbert space and not any actual discrepancy.

Very important criticism was delivered, however, by Einstein and collaborators [14] who proposed the so called EPR Gedankenexperiment to argue that some action at macroscopic distance between microscopic particles should exist in the Copenhagen model. The EPR problem has been repeatedly discussed, which has been accompanied by some important mistakes leading to false conclusions. The whole story and contemporary solution of the given problem will be described in Sec. 5.

However, before ending the present section another criticism should be yet added. It concerns the existence of discrete states in Schrödinger equation when the two last assumptions are added. It is evident that in such a case all mathematical superpositions should represent mutually equivalent physical states and practically no quantized (discrete) states should exist in experimental reality. The given problem has been removed in the HV theory as only eigenstates belonging to Hamiltonian eigenvalues represent now ”pure” states and any superposition of theirs represents statistical mixture.

It means that the structure of the Hilbert space in the HV theory differs significantly from that required in the case of the Copenhagen quantum mechanics. It consists always of mutually orthogonal subspaces and depends on the type of the corresponding physical system (see Sec. 4 and eventually also Sec. 10). The time flow is then described as irreversible, which will be demonstrated on the physical situation corresponding to the evolution of two-particle system being described in the next section.
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4 Hidden-variable theory and irreversible time flow

The HV theory differs from the Copenhagen quantum mechanics also in one further important point; the time evolution is not more reversible, which follows simply from the Schrödinger equation and the extended Hilbert space.

Let us suppose now that the given physical system consists of two particles. It may be described in its center of mass system (CMS) with the help of Hamiltonian

$$ H = \frac{p^2}{2m} + V(x), \quad (9) $$

where \( m \) is the reduced mass, \( p \) - momentum of one particle, \( V(x) \) is the mutual potential between the given particles and \( x \) represents the relative distance between the particles. The Hilbert space \( \mathcal{H} \) corresponding to the HV theory consists then of two subspaces:

$$ \mathcal{H} \equiv \{ \Delta^-, \Delta^+ \}, \quad (10) $$

that are mutually related by the evolution operator

$$ U(t) = e^{-iHt} \quad (t \geq 0). \quad (11) $$

It holds, e.g., (see [10, 11])

$$ \mathcal{H} = \Sigma_t U(t)\Delta^- = \Sigma_t U(-t)\Delta^+. \quad (12) $$

Individual subspaces \( \Delta^- \) and \( \Delta^+ \) are spanned on one set of Hamiltonian eigenfunctions in the usual way. However, the superposition principle cannot be applied to in the physical sense. The basic physical states are represented by Hamiltonian eigenstates only; their superpositions represent statistical combinations of basic states. Any \( t \)-dependent function \( \psi(x, t) \) obtained by solving the Schrödinger equation may be represented by a trajectory characterized by given initial conditions.

In case of continuous Hamiltonian spectrum (two free particles) any point on such a trajectory may be characterized by expectation values of the operator

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where \( q \) and \( p \) are coordinates and momentum components of one particle in CMS. The states belonging to \( \Delta^- \) are incoming states, and those of \( \Delta^+ \) - outgoing states (independently of chosen coordinate system). The evolution goes always in one direction from "in"-subspace to "out"-subspace (for more details see [15]).

As these two different kinds of states represent quite different experimental situations it is necessary to separate "in" and "out" states into two mutually orthogonal subspaces. It is then also possible to join an additional orthogonal subspace that might represent corresponding resonance formed in a particle collision (or an unstable particle decaying into the given particle pair), i.e.

\[
\mathcal{H} \equiv \{ \Delta^- \oplus \Theta \oplus \Delta^+ \}; \tag{13}
\]

see also Ref. [16] where the corresponding extended Hilbert space was derived independently as the consequence of exact exponential decay law. It is only necessary to define the action of evolution operator between \( \Theta \) and other subspaces in agreement with evolution defined already in individual \( \Delta^\pm \)-subspaces. The evolution goes in one direction, at least from the global point of view; evolution processes between internal states of \( \Theta \) may be rather chaotic. However, global trajectories tend always in one direction; see the scheme in Fig. 1.

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**Figure 1:** Scheme of the Hilbert space (for a two-particle system) extended according to original proposal of Lax and Phillips; three mutually orthogonal subspaces and irreversible time evolution.
The subspace $\Theta$ may represent a resonance (or generally an unstable object) decaying into the given particle pair. The unstable object may be, of course, formed also in the decays of heavier unstable objects. In such a case the decay process may be represented always as the transition from $\Theta$ to $\Delta^+$. In the case of discrete Hamiltonian spectrum (e.g., harmonic oscillator) the wave function has similar irreversible $t$-dependent form. However, the evolution is periodical as a rule. The evolution may be again characterized by trajectories corresponding to different initial conditions. The corresponding Hilbert space consists then of infinite number of identical pairs of mutually orthogonal subspaces belonging subsequently to two intervals of phase: $(k.\pi, (k + 1).\pi)$ and $((k + 1).\pi, (k + 2).\pi)$; individual states being characterized by expectation values of phase $\Phi$ lying in principle in the interval from $-\infty$ to $+\infty$ (see also [15]).

Such an extended model enables to represent the time evolution described by Schrödinger equation in the Hilbert space in full agreement with actual behavior of physical systems. It removes fully the criticism of Pauli; however, it does not give any answer to the criticism of Susskind and Glogover. To this goal some other extension of Hilbert space would be necessary as introduced in Sec. 3.

As to the irreversibility of time evolution it corresponds to the behavior of real objects in microscopic world similarly as it has been in macroscopic one. There is not practically any greater physical gap between these two regions.

5 EPR experiment and HV theory

In the early years of quantum mechanics only the Copenhagen alternative was taken as valid since the physical community was influenced strongly by the "proof" of von Neumann [17] that any other (hidden) parameters were excluded by the standard quantum-mechanical mathematical model and that the given quantum-mechanical model represented complete description of microscopic world. It remained without any attention that already in 1935 Grete Hermann [18] showed that the approach of von Neumann was practically a "circle proof".

It was undoubtly the main reason why also the criticism of Einstein and his collaborators [14] was refused. Einstein proposed the
known EPR Gedankenexperiment and argued in 1935 that quantum mechanics was not a complete theory to describe fully a physical system. Bohr [19] opposed strongly; having stated that the Copenhagen model corresponds fully to the microscopic reality. And physical community accepted practically his standpoint.

The argument that a hidden variable was contained already in Schrödinger equation was published by D. Bohm [20] in 1952; however, it was accepted seriously by a very small number of physicists. Only the approach of J. Bell [21] met with greater attention, as also some formulas were presented that seemed to be able to bring the decision between the two discussed (Copenhagen and ensemble) alternatives of quantum-mechanical model on experimental basis.

However, in broad physical community there has not been any interest to change generally accepted paradigm. Any greater doubts about the standard quantum theory have not been evoked, which was influenced from the very beginning in principle by the mentioned mistaking proof of von Neumann.

Nevertheless, the approach initiated by Bell influenced decisively the whole further story of the EPR experiment. The original Gedanken experiment was modified to be possible to perform it. And the corresponding experiments started to be prepared. However, two other mistaking arguments played a decisive role in the solution of the given problem.

One argument followed from the statement of Belinfante [22] that the Copenhagen alternative and the HV theory had to give mutually different predictions, or in other words, that the prediction of HV theory had to differ significantly from the Malus law (approximately measured dependence of light transmission in the case of two polarizers). That was not, however, true as the given statement was based on mistaking interchange of transmission probabilities through a polarizer pair and one polarizer; a more detailed explanation having been given, e.g., in Refs. [23, 24]. In fact, the approximate Malus law (as measured) has been fully consistent with the HV theory.

The other mistaking argument was then involved in the already mentioned formula of Bell, which will be explained to a greater detail in the next section. Now the actual story of the EPR experiment will be described.

The experimentally feasible EPR experiment consisted in the mea-
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measurement of coincidence transmission probabilities of two photons with opposite spins and running in opposite directions through two polarizers:

\[ < - - - | \beta - - - o - - - | \alpha - - - > \]

, where \( \alpha \) and \( \beta \) are deviations of individual polarizer axes from a common zero position. According to Bell [21] any four transmission probabilities \( a_j, b_j \) \((j = 1, 2)\) corresponding to two different orientations of both the polarizers (4 different combinations) should have fulfilled the following condition

\[ B = a_1 b_1 + a_2 b_1 + a_1 b_2 - a_2 b_2 \leq 2 \]

, if the HV theory had hold (which has not been, however, true as it will be shown in Sec. 6). For the Copenhagen alternative this upper limit should have been higher. The then situation was described to a greater detail in already mentioned book of Belinfante [22].

The main series of corresponding EPR experiments was being performed since 1971. It was finished practically in 1982 by the following conclusion (see Aspect et al. [25]):

- Bell’s inequalities have been violated; and the hidden-variable alternative seemed, therefore, to be refused;

- measured values have been practically in agreement with quantum-mechanical predictions (approximate Malus law having been obtained).

At that time the results were interpreted as the victory of Copenhagen quantum mechanics. However, it was seen soon that practically nothing from earlier problems was solved. And the discussions concerning EPR experiments have continued. In fact none of both the results of Aspect’s experiment has led to refusal of HV theory as the corresponding conclusion has been based on the mentioned mistakes.

As to the latter result of Aspect et al. obtained in the given EPR experiment (approximate Malus law) we have already mentioned the mistake of Belinfante and referred to its detailed explanation in Ref. [24]. The approximate Malus law may correspond fully to both theoretical alternatives and no such an argument against the HV theory exists.

More detailed explanation is needed, of course, as to the first experimental result. Bell started in principle from the HV approach.
However, to derive the given inequalities he had to interchange some probabilities, which seemed to be natural at the first sight only. In fact, it represented a strong assumption that corresponded to the passage from the hidden-variable concept to the classical one. The given inequalities were derived, of course, in other ways, too; see, e.g., Ref. [26]. Similar assumptions were involved, however, in all these approaches (see [27]). Bell’s limit corresponded to classical physics and not to HV theory, which will be discussed and explained to a greater detail in the next section.

However, at the end of this section it is necessary to mention also the recent delayed-choice experiment [28] trying to bring a new support for the Copenhagen alternative. It follows from our theoretical analysis that no further experiment of EPR type may influence our conclusions concerning the decisive preference of the hidden-variable theory.

6 Bell’s operator and different inequalities

From the preceding it is evident that Bell’s combination of coincidence probabilities may exhibit different limits according to basic assumptions concerning the individual processes. We will discuss this problem now in the language of the so called Bell operator obtained by substituting individual probabilities by basic operators representing individual measurement acts (see, e.g., [29]). According to chosen assumptions three different limits may be obtained.

The Bell operator $B$ may be represented in the Hilbert space

$$\mathcal{H} = \mathcal{H}_a \otimes \mathcal{H}_b,$$

(14)

where the subspaces $\mathcal{H}_a$ and $\mathcal{H}_b$ correspond to individual measuring devices (polarizers) in the coincidence arrangement. It is then possible to introduce the operator

$$B = a_1 b_1 + a_1 b_2 + a_2 b_1 - a_2 b_2,$$

(15)

where $a_j$ and $b_k$ are now operators acting in subspaces $\mathcal{H}_a$ and $\mathcal{H}_b$ and corresponding to different measurements in individual polarizers. It holds for the expectation values of these operators [29]

$$0 \leq |\langle a_j \rangle|, |\langle b_k \rangle| \leq 1.$$
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The expectation values $|\langle B \rangle|$ of the Bell operator may then possess different upper limits according to the mutual commutation relations of the operators $a_j$ and $b_k$.

If it holds

$$[a_1, a_2] \neq 0, \quad [b_1, b_2] \neq 0,$$

$$[a_j, b_k] \neq 0, \quad [a_1, a_2] \neq 0, \quad [b_1, b_2] \neq 0,$$

one can obtain by a rough estimate [30]

$$\langle BB^+ \rangle \leq 16 \quad \text{or} \quad \langle B \rangle \leq 4.$$  

However, after more detailed calculation one obtains (see [31])

$$\langle BB^+ \rangle \leq 12, \quad |\langle B \rangle| \leq 2\sqrt{3}. \quad (16)$$

If

$$[a_j, b_k] = 0, \quad \text{and} \quad [a_1, a_2] \neq 0, \quad [b_1, b_2] \neq 0,$$

it holds

$$\langle BB^+ \rangle \leq 8, \quad |\langle B \rangle| \leq 2\sqrt{2}. \quad (17)$$

And finally, if all operators $a_j$ and $b_k$ commute mutually one obtains

$$\langle BB^+ \rangle \leq 4, \quad |\langle B \rangle| \leq 2; \quad (18)$$

the same limit being obtained also if at least the operators belonging to one of subspaces $\mathcal{H}_a$ or $\mathcal{H}_b$ commute mutually and with all other operators [31].

It has been, therefore, derived that there are in principle three different limiting bounds for Bell’s combination of coincidence probability measurements. In the following we will attempt to correlate them to individual physical alternatives:

(i) In contradistinction to hitherto common opinion the last limit (18) corresponds to the conditions of classical physics.

(ii) The limit (17) represents the properties of the HV alternative. There are not, of course, any hidden parameters; all physical parameters are standard ones; only some of them being statistically distributed in corresponding initial states.
(iii) As to the limit (16) it represents the case when the results of both the measuring devices may influence mutually one another; it would be the case of the orthodox (Copenhagen) quantum mechanics.

It is only the classical limit that has been excluded by experimental EPR data. As to the HV alternative it does not contradict the experimental results and may be brought to agreement with experimentally established coincidence polarization data (obtained, e.g., by Aspect et al. [25]). It is, of course, also the Copenhagen quantum mechanics that has not been excluded by EPR experiment results. Here the previously discussed internal discrepancies must be taken into account.

7 HV theory and physical reality

It follows from the preceding that there are not any objections against the HV theory as to the description of microscopic world. However, the introduced arguments indicate that this theory should be not only preferred to the standard quantum mechanics but also used in describing all physical reality, as it has been considered recently by A. Legget [32, 33]. He has asked whether the every day world may be described with the help of the same physical model as microscopic objects. However, he has not taken into account the significant difference between two alternatives of quantum-theoretical model: statistical (or HV theory) and Copenhagen. Consequently, he has not been able to get a positive answer. His question has been answered, of course, positively in our paper, as the former alternative, i.e. HV theoretical approach, differs from the classical physics only in the existence of discrete energy spectrum in closed physical systems (see also Sec. 9).

The HV theory may be applied, therefore, practically also to the description of standard macroscopic processes, since in the case of discrete spectrum the differences between individual energy values should remain quite immeasurable. However, quite recent analyses concerning the cavity optomechanical systems seem to provide the way to much stronger support for the HV theory to be denoted as generally valid; see e.g. [34, 35]. The behavior analogous to the excitations of single molecules may be expected for pairs of membranes in vibrational states [35]. However, it is not possible to interpret
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such states as the phenomenon of entanglement (see Ref. [36]) as this phenomenon might be bound only to the Copenhagen quantum mechanics that contains internal contradictions. The application of the HV theory might be probably very helpful in the solution of corresponding problems of optomechanics.

It is also the famous problem of Schrödinger cat (representing a macroscopic closed physical system) that may be seen in a quite new light now. There are two basic states ("eigenstates"), i.e., living and dead cat, and any superposition of these states correspond to their statistical combination.

8 Light transmission through three polarizers

In the preceding we have discussed the internal discrepancies and unphysical aspects of the Copenhagen quantum mechanics and demonstrated that they might be removed by passing to the HV theory. At the same time the criticism of Einstein may be removed, too. Any objections against HV theory do not exist.

In the case of the EPR experiment with polarized photons both quantum theories may give practically the same predictions for coincidence transmission, even if the physical interpretations of polarization and transmission mechanisms are quite different. And one may ask whether different predictions may exist for another arrangement of polarizers. Already some time ago we have attempted to analyze the transmission of light through three polarizers:

\[ o - - - | - - - |^{\alpha} - - - |^{\beta} - - - > \]

where individual polarizers have been differently oriented; \( \alpha \) and \( \beta \) denoting axis deviations of the second and third polarizers from the first one. According to the Copenhagen quantum-mechanical model it should hold for corresponding light transmission probability

\[ P(\alpha, \beta) = \cos^2 \alpha \cos^2 (\alpha - \beta). \] (19)

And it was possible to expect measurable deviations in the case of the HV theory.

The corresponding experiment was performed and the results were published in 1993-4; see [37, 38]. For a given angle \( \alpha \) the angle \( \beta \) was
always established, so as the total light transmission be minimal. Fundamental deviations from the given quantum-mechanical formula (19) have been found, as it may be seen from Figs. 2 and 3.

The mentioned angle pairs (giving the minimum light transmission for given $\alpha$) are shown in Fig. 2. And the corresponding experimental values of transmitted light are represented by dashed line in Fig. 3 (experimental points taken from [38]). The quantum-mechanical predictions for the given angle pairs are then represented by full line; the position of this line being shifted in vertical direction somewhat arbitrarily as the values of the so called ”imperfectness” of given polarizers were not available. In any case the standard quantum-mechanical prediction requires maxima at the positions where the experiment exhibits deep minima. The difference against the Copenhagen quantum mechanics was not accented in Ref. [38] where the experimental results were published for the first time as we were afraid reasonably that the paper would not have been accepted for publication in such a case. We have only mentioned that similar characteristics may be obtained in the framework of classical theory of Stokes.

And it is possible to conclude that the given experimental data represent further falsification of Copenhagen quantum mechanics, while good agreement may be evidently obtained in the framework of HV theory as a series of free parameters for the polarization description is available.

9 Schrödinger equation and classical physics

To complete our analysis it is necessary to discuss the actual relation between the Schrödinger equation and classical physics. E. Schrödinger [5] was successful with his equation when he showed that for particles exhibiting inertial motion the identical behavior with classical physics was obtained. Let us return now, therefore, to the time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = H\psi(x,t),$$  \hspace{1cm} (20)

where the complex function $\psi(x,t)$ is expressed as

$$\psi(x,t) = \lambda(x,t)e^{i\frac{\hbar}{\lambda} \Phi(x,t)}$$  \hspace{1cm} (21)
and both the functions $\lambda(x, t)$ and $\Phi(x, t)$ are real. Let us limit to time-independent potential $V(x)$ (see Eq. (1)). Eq. (20) may be substituted by two equations for two real functions (see D. Bohm [20])

$$\frac{\left(\nabla \Phi\right)^2}{2m} + V(x) + V_q(x, t) = -\partial_t \Phi,$$

$$\Delta \Phi + 2(\nabla \Phi)(\nabla \log \lambda) = -2m \partial_t \log \lambda,$$

where

$$V_q(x, t) = -\frac{\hbar^2}{2m} \frac{\Delta \lambda}{\lambda}$$

has been denoted as quantum potential.

Eq. (22) resembles Hamilton-Jacobi equation

$$\frac{1}{2m} (\nabla S(x, t))^2 + V(x) = -\partial_t S(x, t),$$

where $S(x, t)$ has been replaced by $\Phi(x, t)$ and the quantum potential $V_q(x, t)$ has been added. $S(x, t)$ is Hamilton principal function, from which the momentum values may be derived:

$$p(x, t) = \nabla S(x, t).$$

For inertial motion it holds

$$V_q(x, t) = V(x) = 0$$

and

$$\Phi(x, t) = S(x, t);$$

the phase being identical with Hamilton principal function in such a case.

Let us assume now

$$V(x) \neq 0$$

and let us limit to basic solutions corresponding to different values of energy (Hamiltonian eigenvalues) and fulfilling the conditions

$$\psi^{(E)}(x, t) = \psi_E(x)e^{-iEt}, \quad H\psi_E(x) = E\psi_E(x).$$

In such a case $V_q(x) \neq 0$ is independent of $t$ and $\Phi(x, t)$ and $S(x, t)$ are mutually different; $V_q(x)$ representing the numerical measure of
such difference. There is not, however, any difference between the physical results of Schrödinger equation and classical physics; see also [2]. All basic solutions of Schrödinger equation are fully equivalent to classical solutions corresponding to the same energy. However, it does not hold in opposite direction. For some solutions of Hamilton equation the corresponding counterparts in the Schrödinger equation (or in the HV theory) do not seem to exist in the case of discrete spectrum.

In the past when the existence of quantum potential was assumed to represent decisive physical difference between Schrödinger equation and classical physics there were done some attempts to interpret it as the consequence of Brown motions of individual microscopic objects. Our result is, however, in full agreement with the results of Ioanidou [39] and Hoyer [40] who have shown that Schrödinger equation may be derived if classical physics is combined with a kind of statistical distribution.

The advantage of the Schrödinger equation consists then in obtaining a complete statistical result in one solution if a statistical distribution of initial basic states is given. Consequently, the Schrödinger equation is very suitable if some initial parameters (e.g., the impact parameter in collision processes and, consequently, also interaction energy values) are not exactly defined and only their statistical distributions may be established, as it occurs in collision measurement approaches.

10 HV theory and Hilbert space

It follows from the preceding that individual solutions of Schrödinger equation may be truly represented in the Hilbert space that is extended (i.e., at least doubled) in comparison to the third assumption introduced in Sec. 2; and when the fourth assumption is refused, too. The evolution of a physical system is then characterized by a trajectory that represents irreversible behavior. Exact solutions may be derived, of course, practically in the case of a system consisting of two stable particles. However, the representation of time evolution in a corresponding Hilbert space may be very helpful also in the case of more complex physical systems.

Our considerations will be based on the analysis of a two-particle system as it has been described in Sec. 4. Such a scheme may repre-
sent a basic structure for describing the evolution of any more complex physical system even if it involves objects that are not stable. A resonance may be formed in the collision of two simpler particles and also the decay of an unstable object (resonance) may be interpreted at least in the first approximation as the transition to two-particle system even if any of the arising particles may be unstable. In such a case the Hilbert space must be, of course, more complex consisting not only of orthogonal sums but also of tensor products of simpler subspaces.

It is clear that all subspaces in one orthogonal sum \((\Delta^+, \Delta^-, \Theta)\) must correspond to the same quantum numbers that must be conserved during the whole evolution. One two-particle system at a given time may be then represented in \(\Delta^{\pm}\) by one vector. However, even the stable objects exhibit usually some internal structures and internal evolutions that might influence the transition probabilities to other subspaces in collision processes. It may be taken into account by substituting, e.g., \(\Delta^-\) by the tensor product of Hilbert subspaces \(\Delta^- \otimes \mathcal{P}_1 \otimes \mathcal{P}_2\) where the two latter subspaces represent main properties of individual objects.

It would be, of course, difficult to allocate a more general Schrödinger equation to such a physical system. In fact, it is hardly possible to describe the detailed evolution of corresponding collision processes when the detailed internal structures and evolutions of individual particles are not known. It is possible to characterize the influence of some characteristics only by establishing some probability distributions in transmission processes. And in such a case the representations of \(\mathcal{P}^{\pm}\) may consist in characterizing them by some basis vectors corresponding to different classes of particle properties that may change during the time evolution. And it is evident that even rather complicated processes might be represented correspondingly (at least at not very high energies).

The same holds before all also for the subspace \(\Theta\) representing a resonance (or an unstable object); it may be suitable to represent it, e.g., by a vector basis corresponding to individual decay channels (see [41]), where the frequencies of individual basic states may be derived from experimental transmission data.
11 Conclusion

The physics of microscopic world in the 20th century has been represented by the Copenhagen quantum mechanics with its logical paradoxes and contradictions. The given theory was taken as valid during the whole century even when Pauli called the attention to one important contradiction already in 1933: the Hamiltonian had to possess the continuous spectrum in the whole interval of all real numbers \((-\infty, +\infty)\) while the actual energy value must be practically positive. Other critical arguments including also the disagreement with experimental data (light transmission through three polarizers) have been then summarized in the preceding.

It has been shown that all known critical points can be removed when one passes from the Copenhagen quantum mechanics to the HV theory that is based practically on mere Schrödinger equation, while earlier additional assumptions (see Sec. 2) are abandoned and the Hilbert space is adapted to actual time-dependent Schrödinger solutions. E.g., for the system of two particles it has been necessary to extend the Hilbert space so as to consist at least of two mutually orthogonal subspaces (see Sec. 4).

The given passage to the HV theory has solved practically all known problems; the mistaking statements having been repaired. It has been also the EPR experiment that should be (and may be) interpreted as it was required by Einstein. It has been shown also that the Schrödinger equation or the HV theory is practically equivalent to classical physics with the only exception, concerning the existence of discrete bound states.

The HV theory may be then applied in principle to the description of microscopic as well as macroscopic objects. There is practically no gap between these two physical regions. It is possible also to say that the question put by A. Legget (see Ref. [33]) has been answered in positive way only if one has passed from Copenhagen quantum mechanics to HV theory (see also Sec. 7).

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GENERALIZATION OF RELATIVISTIC PARTICLE DYNAMICS ON THE CASE OF NON-RIEMANNIAN SPACE-TIME GEOMETRY

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Abstract

Conventional relativistic dynamics of a pointlike particle is generalized on the case of arbitrary non-Riemannian space-time geometry. Non-Riemannian geometry is an arbitrary physical geometry, i.e. a geometry, described completely by the world function of the space-time geometry. The physical geometry may be discrete, or continuous. It may be granular (partly continuous and partly discrete). As a rule the non-Riemannian geometry is nonaxiomatizable, because the equivalence relation is intransitive. The dynamic equations are the difference equations. They do not contain references to a dimension and to a coordinate system. The generalization is produced on the dynamics of composite particles, which may be identified with elementary particles. The granular space-time geometry generates multivariant motion, which is
responsible for quantum effects. It generates a discrimination mechanism, which is responsible for discrete values of the elementary particles parameters. The quantum principles appear to be needless in such a dynamics.
1 Introduction

This paper is devoted to expanding of relativistic dynamics on the case of the non-Riemannian space-time geometry. It is a generalization of classical principles on the case of more general space-time geometries. Necessity of such a generalization appeared, when it became to be clear, that the space-time geometry may be not only a Riemannian one. The space-time geometry may be a physical geometry, i.e. a geometry, which is described by its world function completely [1, 2]. Another name of the physical geometry is the tubular geometry (or T-geometry). In general, straight lines in the tubular geometry are tubes (surfaces), but not one-dimensional lines.

In the approximation, when the influence of the matter on the space-time geometry is neglected, the space-time geometry is to be equal at all points of the space-time. It means, that the geometry is uniform and isotropic. In the class of Riemannian geometries there is only one uniform and isotropic geometry (appropriate for the space-time description): the geometry of Minkowski. In the class of physical geometries any geometry \( G \), described by the world function

\[
\sigma = F(\sigma_M), \quad F(0) = 0
\]

is isotropic and uniform. Here

\[
\sigma_M(x, x') = \frac{1}{2} g_{ik}(x^i - x'^i)(x^k - x'^k), \quad g_{ik} = \text{const}
\]

is the world function of the geometry of Minkowski and \( F \) is an arbitrary function.

As far as the world function \( \sigma_M \) is invariant with respect to Lorentz transformations, any function (1) of \( \sigma_M \) is also invariant. Then the physical geometry, described by the world function of the form (1) is also Lorentz-invariant.

The classical principles of dynamic do not work in the microcosm, provided one uses the space-time geometry of Minkowski. It is supposed conventionally, that in the microcosm the principles of quantum dynamics are valid. We shall refer to such an approach (quantum principles of dynamics + geometry of Minkowski) as the quantum paradigm.

However, another approach (classical principles of dynamics + some space-time geometry of the form (1)) is possible also. This
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approach will be referred to as a geometrical paradigm. In the framework of the geometrical paradigm the space-time geometry is not determined. It must be chosen inside the class of physical geometries in such way, that the classical particle dynamics, based of the chosen space-time geometry, explains all experimental data in the microcosm physics. Thus, in the framework of the geometrical paradigm the classical principles of dynamics are fixed, whereas the space-time geometry is varied.

On the contrary, in the framework of quantum paradigm the space-time geometry is fixed, and principles of dynamics are varied. It is evident, that from a technical viewpoint a variation of dynamic principles is more complicated, than a variation of the space-time geometry, which is determined by the world function completely. Besides, the quantum paradigm is generated by our poor knowledge of geometry, whereas the geometrical paradigm takes into account our more perfect knowledge of geometry. Let me explain the situation in a simple example.

Let us imagine a person N (Nicola), who does not know, that the algebraic equation may have many roots. (He thinks, that the algebraic equation has only one root). Such a situation seems to be fantastic, but nevertheless, let us consider this situation. Nicola is a physicist-theorist, who creates fundamental physical theories. In general, Nicola does know algebra. He can add and multiply the numbers, he can even differentiate and integrate. In other words, he possesses the contemporary mathematical technique. However, Nicola does not know, that the algebraic equation may have many roots. Constructing theories, Nicola may meet such a situation, when one needs to use several roots of an algebraic equation. In this case he should think about his knowledge of algebra.

But Nicola is self-opinionated. He does not question his knowledge of algebra. He invents new hypotheses, which admit him to compensate his poor knowledge of algebra. These new hypotheses are simple fittings, but in some cases these fittings work successfully. In other situations these fittings cease to work, and Nicola is forced to search for other fittings (hypotheses).

Let us imagine Mike, who is a friend of Nicola. Mike knows very well, that the algebraic equation may have many roots, and he says to Nicola: "Nicola, try to use the fact, that the algebraic equation
may have many roots. Maybe, some problems of your fundamental physical theory disappear by themselves. You will not need to invent your hypotheses, and your theory will contain less fundamental principles”. Nicola exclaims: ”Oh, Mike! This is a splendid idea! One needs to test this hypothesis! However, it is your idea. Test it yourself, please. If it will appear, that by means of your idea one can explain more experiments, than I can explain by my hypotheses, we create a new fundamental physical theory. However, if your idea helps to explain only those experiments, which are explained by my theory, I do not see any reason to use your idea in the fundamental physical theory, because my theory explains all known experimental data. Go ahead! Introduce your idea in the theory and test it!”

It remains to add, that Mike has suggested only to use mathematics correctly and nothing besides. Nicola perceived Mike’s suggestion as a new theoretical conception. The presented story seems to be fantastic, however, the situation, when one chooses the geometry of Minkowski among many possible uniform isotropic geometries is a mathematical mistake of the same kind, as Nicola has made. This mistake is a source of the quantum paradigm. One may say, that we did not know another uniform isotropic geometries other, than the geometry of Minkowski. It is true. However, it is a mathematical mistake, which should be corrected.

Reaction of Nicola is a typical reaction of contemporary theorist, which does not distinguish between a hypothesis and a correction of a mistake, based on our poor knowledge of mathematics. Most contemporary theorists believe that a fundamental physical theory is a list of correctly solved problems, and the physics of microcosm progresses by means of invention of new physical hypotheses. They did not understand, that the fundamental theory deals with physical principles, but not with single physical phenomena. They understand the role of physical principles in usual physics. However, as it concerns the microcosm physical phenomena, they believe, that usual classical principles of physics are insufficient in the description of the microcosm physics, and the theorists retire from the classical principles. In other words, they believe in the quantum paradigm, founded on our poor knowledge of geometry.

Working in the framework of the geometrical paradigm, we are to choose the true space-time geometry in the microcosm. This choice
is rather complicated, because in the true space-time geometry the dynamics of particle is to explain all experimental data. To test coincidence of the theory predictions with experiment, one is to construct particle dynamics in any possible space-time geometry. In reality, it is a generalization of the particle dynamics in the Riemannian space-time geometry (which is known) on the case of the general physical space-time geometry (which is not yet known). This problem is solved in the present paper.

Let us stress, that producing such a generalization, we shall not test our results by means of a comparison with the experiment. Such a comparison is impossible and useless, if we work with arbitrary space-time geometry (but not with the geometry, which is supposed to be a true space-time geometry). In order to find the true space-time geometry of microcosm, one needs to formulate principles of classical dynamics in arbitrary physical space-time geometry and to construct corresponding mathematical formalism. In this paper we construct the mathematical formalism of dynamics. However, we do not try to choose the true space-time geometry of microcosm. Concrete space-time geometries, considered in the paper are used only as an illustration of capacities of the geometrical paradigm. They do not pretend to be an example of a true space-time geometry.

In the end of nineteenth century the physics developed in the direction of its geometrization, i.e. the more properties of physical phenomena were explained by properties of the event space (space-time). Explanation of the conservation laws by means of isotropy and homogeneity of the event space, the special relativity, the general relativity, explanation of the electric charge discreteness by compactification of 5-dimensional Kaluza-Klein geometry are consequent stages of the physics geometrization. Geometrization of physics was a very effective program of the theoretical physics development.

However, attempts of this program applications to the microcosm physical phenomena failed. This failure was conditioned by the very sad circumstance, that our knowledge of geometry were poor. We could describe only continuous geometries with unlimited divisibility. We could not work with granular geometries, i.e. with geometries, which are partly continuous and partly discrete. We did not know, how one can describe a geometry with limited divisibility. We could not imagine, that there are multivariant geometries, where at the
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point $P_0$ there exist many vectors $P_0P_1, P_0P_2, P_0P_3, \ldots$, which are equivalent to the vector $Q_0Q_1$ at the point $Q_0$, but these vectors $P_0P_1, P_0P_2, P_0P_3, \ldots$ are not equivalent between themselves. We could not imagine, that the geometry in itself may discriminate existence of some geometrical objects. In reality the space-time geometry of microcosm possessed such exotic properties, however we could not describe these properties. Our knowledge of geometry were too poor. However, the multivariance [3] is a very important property of the space-time geometry, which is responsible for quantum effects [4].

All generalized geometries are modifications of the proper Euclidean geometry, constructed by Euclid many years ago. Euclid presented two very important matters: (1) the Euclidean geometry and (2) the Euclidean method of the geometry construction.

Conventionally one uses the Euclidean method for construction of generalized geometries. This method is only a half-finished product (the product is the Euclidean geometry itself). Using the Euclidean method, one can construct only axiomatizable geometries. The axiomatizable geometries are such geometries, where all geometrical objects can be constructed of blocks. Euclid himself used three kinds of such blocks: point, segment of straight and angle. Formalization of the construction procedure leads to the statement, that all propositions of the proper Euclidean geometry may be deduced from a finite system of axioms. One supposes, that for construction of a generalized geometry one has to use another system of axioms (i.e. the Euclidean blocks are to be replaced by another series of blocks). Thus, the Euclidean method admits one to construct only axiomatizable geometries.

Another method of the generalized geometry construction admits one to construct only physical geometries, i.e. geometries, which can be described completely by the world function of the geometry in question. The world function $\sigma$ is defined by the relation $\sigma(P,Q) = \frac{1}{2} \rho^2(P,Q)$, where $\rho(P,Q)$ is the distance between the points $P$ and $Q$. This method uses the already constructed proper Euclidean geometry as a standard geometry. The proper Euclidean geometry $G_E$ is a physical geometry. All propositions $\mathcal{P}$ of the proper Euclidean geometry $G_E$ are presented in the form $\mathcal{P}(\sigma_E)$, where $\sigma_E$ is the world function of $G_E$. Thereafter one deforms the standard geometry $G_E$, replacing $\sigma_E$ by the world function $\sigma$ of some other
physical geometry \( \mathcal{G} \): \( \mathcal{P}(\sigma_E) \rightarrow \mathcal{P}(\sigma) \). One obtains the set \( \mathcal{P}(\sigma) \) of all propositions of the physical geometry \( \mathcal{G} \). The physical geometry \( \mathcal{G} \), obtained from the standard (proper Euclidean) geometry by means of the deformation is not an axiomatizable geometry, in general, i.e. it cannot be constructed of any blocks.

Let us demonstrate this fact in a simple model. Let we have only one kind of cubic plasticine blocks. These blocks are painted by a red paint, in order one can distinguish boundaries of blocks in a building. Let us construct some building of these blocks, for instance, a cube. Let us deform this cube in an arbitrary way, for instance, into a circular cylinder. After such a deformation all cubic blocks, constituting the cube will be deformed. The deformation will be different for different blocks, and they cannot be used for construction of a new building. Of course, one can reconstruct the cylinder, but this cylinder will be reconstructed of blocks, having different shapes, which they have been obtained as a result of the deformation. These blocks are not suitable for construction of another buildings. This model shows, how a deformation destroys the axiomatizability of the axiomatizable geometry.

In any axiomatizable geometry the equivalence relation is transitive. This transitivity is necessary, in order that any deduction leads to a definite result. Deformation destroys the transitivity of the equivalence relation, and the geometry becomes to be nonaxiomatizable. Let us demonstrate this in the example of two vector equivalence. In the proper Euclidean geometry \( \mathcal{G}_E \) the equivalence of two vectors \( \mathbf{P}_0 \mathbf{P}_1 \) and \( \mathbf{Q}_0 \mathbf{Q}_1 \) is defined as follows. Vectors \( \mathbf{P}_0 \mathbf{P}_1 \) and \( \mathbf{Q}_0 \mathbf{Q}_1 \) are equivalent \( (\mathbf{P}_0 \mathbf{P}_1 \equiv \mathbf{Q}_0 \mathbf{Q}_1) \), if vectors \( \mathbf{P}_0 \mathbf{P}_1 \) and \( \mathbf{Q}_0 \mathbf{Q}_1 \) are in parallel \( (\mathbf{P}_0 \mathbf{P}_1 \uparrow \downarrow \mathbf{Q}_0 \mathbf{Q}_1) \) and their lengths \( |\mathbf{P}_0 \mathbf{P}_1| \) and \( |\mathbf{Q}_0 \mathbf{Q}_1| \) are equal. Mathematically these two conditions are written in the form
\[
(\mathbf{P}_0 \mathbf{P}_1 \uparrow \downarrow \mathbf{Q}_0 \mathbf{Q}_1) : \quad (\mathbf{P}_0 \mathbf{P}_1 \cdot \mathbf{Q}_0 \mathbf{Q}_1) = |\mathbf{P}_0 \mathbf{P}_1| \cdot |\mathbf{Q}_0 \mathbf{Q}_1| \quad (3)
\]
\[
|\mathbf{P}_0 \mathbf{P}_1| = |\mathbf{Q}_0 \mathbf{Q}_1|, \quad |\mathbf{P}_0 \mathbf{P}_1| = \sqrt{2\sigma(\mathbf{P}_0, \mathbf{P}_1)} \quad (4)
\]
where \( (\mathbf{P}_0 \mathbf{P}_1 \cdot \mathbf{Q}_0 \mathbf{Q}_1) \) is the scalar product of two vectors, defined by the relation
\[
(\mathbf{P}_0 \mathbf{P}_1 \cdot \mathbf{Q}_0 \mathbf{Q}_1) = \sigma(\mathbf{P}_0, \mathbf{Q}_1) + \sigma(\mathbf{P}_1, \mathbf{Q}_0) - \sigma(\mathbf{P}_0, \mathbf{Q}_0) - \sigma(\mathbf{P}_1, \mathbf{Q}_1) \quad (5)
\]
Here $\sigma$ is the world function of the proper Euclidean geometry $G_E$. The length $|PQ|$ of vector $PQ$ is defined by the relation

$$|PQ| = \rho(P, Q) = \sqrt{2\sigma(P, Q)}$$

(6)

Using relations (3) - (6), one can write the equivalence condition in the form

$$P_0P_1\equiv Q_0Q_1 : \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) = \sigma(P_0, P_1) \land \sigma(P_0, P_1) = \sigma(Q_0, Q_1)$$

(7)

The equivalence relation is used in any physical geometry. The definition of equivalence (7) is a satisfactory geometrical definition, because it does not contain a reference to the dimension of the space and to the coordinate system. It contains only points $P_0, P_1, Q_0, Q_1$, determining vectors $P_0P_1$ and $Q_0Q_1$ and distances (world functions) between these points. The definition of equivalence (7) coincides with the conventional definition of two vectors equivalence in the proper Euclidean geometry. If one fixes points $P_0, P_1, Q_0$ in the relations (7) and solve these equations with respect to the point $Q_1$, one finds that these equations always have one and only one solution. This statement follows from the properties of the world function of the proper Euclidean geometry. It means that the proper Euclidean geometry is single-variant with respect any pairs of its points. It means also, that the equivalence relation is transitive in the proper Euclidean geometry. By definition the transitivity of the equivalence relation means, that

if $P_0P_1\equiv Q_0Q_1 \land Q_0Q_1\equiv R_0R_1$, then $P_0P_1\equiv R_0R_1$ (8)

In the arbitrary physical geometry the equivalence relation has the same form (7) with another world function $\sigma$, satisfying the constraints

$$\sigma : \Omega \times \Omega \rightarrow \mathbb{R}, \quad \sigma(P, P) = 0, \quad \forall P, Q \in \Omega$$

(9)

Here $\Omega$ is the set of all points, where the geometry is given.

In the case of arbitrary world function one cannot guarantee, that equations (7) have always a unique solution. There may be many
solutions. In this case one has a multivariant geometry. There may be no solution. In this case one has a zero-variant (discriminating) geometry. In both cases the equivalence relation is intransitive, and geometry is nonaxiomatizable. There may be such a situation, that the geometry is multivariant with respect to some points and vectors, and it is zero-variant with respect to other points and vectors. Such a geometry will be also qualified as a multivariant geometry.

One sets conventionally, that the world function of the space-time is symmetric

$$\sigma(P, Q) = \sigma(Q, P), \quad \forall P, Q \in \Omega$$  \hspace{1cm} (10)

This condition means that the future and the past are geometrically equivalent. However, the physical geometry can be constructed for asymmetric world function $\Sigma$ [5]

$$\Sigma(P, Q) = G(P, Q) + A(P, Q)$$  \hspace{1cm} (11)

$$G(P, Q) = G(Q, P), \quad A(P, Q) = -A(Q, P)$$  \hspace{1cm} (12)

The time is considered as an attribute of the event space (space-time). The time arrow can be taken into account in the technique of asymmetric space-time geometry.

The asymmetric geometry with asymmetric world function may appear in the microcosm, however, its application is especially interesting in the cosmology, where the future and the past of our universe may appear to be not equal. Besides, the gravitational law in asymmetric space-time geometry distinguishes from the gravitational law in the symmetric one. Maybe, reasonable supposition on asymmetry of the space-time geometry will be able to explain deflection of astronomical observations from predictions of the general relativity. In this case the invention of the dark matter may appear to be needless. However, such a possibility is not yet investigated properly.

The granular space-time geometry $G_g$, given on the manifold of Minkowski is described approximately by the world function $\sigma_g$

$$\sigma_g = \sigma_M + \lambda_0^2 \left\{ \begin{array}{ll} \text{sgn} (\sigma_M) & \text{if} \ |\sigma_M| > \sigma_0 \\ \sigma_M \sigma_0 & \text{if} \ |\sigma_M| \leq \sigma_0 \end{array} \right., \quad \lambda_0^2, \sigma_0 = \text{const} \geq 0$$  \hspace{1cm} (13)
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where \( \sigma_M \) is the world function of the geometry \( \mathcal{G}_M \) of Minkowski, \( \lambda_0 \) is a elementary length. The world function \( \sigma_M \) of the Minkowski geometry \( \mathcal{G}_M \) is Lorentz-invariant, and the world function \( \sigma_g \) of the granular geometry \( \mathcal{G}_g \) is Lorentz-invariant also, because it is a function of \( \sigma_M \). If \( \sigma_0 = 0 \), the geometry \( \mathcal{G}_g \) is discrete, although it is given on the continuous manifold of Minkowski. Indeed, if \( \sigma_0 = 0 \), in the geometry \( \mathcal{G}_g \) there are no close points separated by a distance less, than \( \sqrt{2} \lambda_0 \). This statement follows from (13). Discrete Lorentz-invariant geometry on a continuous manifold! This fact seems to be very unexpected at the conventional approach to geometry, where discreteness of geometry depends on the structure of the point set \( \Omega \), where the geometry is given, and where the geometry is formulated in some coordinate system.

In the physical geometry a discreteness and a continuity of the geometry are determined by the world function and only by the world function, whereas the structure of the point set \( \Omega \) is important only in such extent, in which it influences on the world function.

Granularity of the geometry \( \mathcal{G}_g \) becomes more clear, if one considers the relative density \( \rho(\sigma_g) = \frac{d\sigma_M(\sigma_g)}{d\sigma_g} \) of points in \( \mathcal{G}_M \) with respect to the density of points in \( \mathcal{G}_g \). Such a density can be introduced, if both geometries \( \mathcal{G}_g \) and \( \mathcal{G}_M \) are uniform, and \( \sigma_g \) is a function of \( \sigma_M \). One obtains from (13)

\[
\rho(\sigma_g) = \frac{d\sigma_M(\sigma_g)}{d\sigma_g} = \begin{cases} 
1 & \text{if } |\sigma_g| > \sigma_0 + \lambda_0^2 \\
\frac{\sigma_0}{\sigma_0 + \lambda_0^2} & \text{if } |\sigma_g| \leq \sigma_0 + \lambda_0^2 
\end{cases} \tag{14}
\]

One can see from (14), that at \( \sigma_0 = 0 \) there is no points in the interval \( \sigma_g \in (-\lambda_0^2, \lambda_0^2) \). It means, that in the case \( \sigma_0 = 0 \) the space-time geometry is discrete. If \( \sigma_0 \neq 0 \), one can see from (14), that the relative density of points in the interval \( \sigma_g \in (-\lambda_0^2 - \sigma_0, \lambda_0^2 + \sigma_0) \) is less, than unity but it is not equal to zero. We have some intermediate situation between the continuity (when \( \rho = 1 \)) and discreteness (when \( \rho = 0 \)). Such a situation is treated as a granularity. Note, that the geometry \( \mathcal{G}_M \) may be considered as a special case of the granular geometry.

In the granular space-time geometry the elementary particle is described by its skeleton \( \mathcal{P}_n = \{P_0, P_1, ..., P_n\} \), consisting of \( n+1 \) points. The pointlike particle is described by the skeleton \( \mathcal{P}_1 = \{P_0, P_1\}, \)

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consisting of two points, or by the vector $P_0P_1$. The vector $P_0P_1$ represents the momentum of the pointlike particle, whereas its length $|P_0P_1| = \mu$ is the geometrical mass of the pointlike particle. The geometrical mass $\mu$ is connected with its usual mass $m$ by means of the relation

$$m = b\mu$$

where $b$ is some universal constant.

Evolution of the elementary particle is described by the world chain, consisting of connected skeletons $P_n^{(0)}, P_n^{(1)}, ..., P_n^{(s)}$...

$$P_n^{(s)} = \{ P_0^{(s)}, P_1^{(s)}, ..., P_n^{(s)} \}, \quad s = 0, 1, ...$$

(16)

The adjacent skeletons $P_n^{(s)}, P_n^{(s+1)}$ of the chain are connected by the relations $P_1^{(s)} = P_0^{(s+1)}, s = 0, 1, ...$ The vector $P_0^{(s)}P_1^{(s)} = P_0^{(s)}P_0^{(s+1)}$ is the leading vector, which determined the world chain direction.

Dynamics of free elementary particle is determined by the relations

$$P_n^{(s)}eqvP_n^{(s+1)}: \quad P_i^{(s)}P_k^{(s)}eqvP_i^{(s+1)}P_k^{(s+1)},$$

$$i, k = 0, 1, ..., n; \quad s = 0, 1, ...$$

(17)

which describe equivalence of adjacent skeletons.

Thus, dynamics of a free elementary particle is described by a system of algebraic equations (17). Specific of dynamics depends on the elementary particle structure (disposition of particles inside the skeleton) and on the space-time geometry.

In the simplest case, when the space-time geometry is the 5-dimensional Kaluza-Klein geometry [6, 7], the dynamic equations (17) for the pointlike particle are reduced to conventional differential dynamic equations, describing motion of the charged pointlike particle in the given electromagnetic and gravitational fields. Thus, dynamic equations (17) can be considered as a generalization of classical differential dynamic equations for the particle motion on the case of the granular space-time geometry. It is a very important fact, which shows, that description of free particles by means of a world chain, consisting of connected skeletons, is simply a generalization of conventional relativistic dynamics of particles, which do not interact.
between themselves. This generalization does not contain any new principles. It is simply a generalization of the particle dynamics onto the case of the granular space-time geometry.

Formally it is a dynamics of free particles, moving in a very deformed and curved space-time geometry. However, dynamics of free particles can be considered as a motion of particles, interacting with some given fields (electromagnetic, gravitational and others) in the space-time geometry of Minkowski. In other words, the motion of a charged particle in the given gravitational and electromagnetic and some other fields can be considered as a motion of a free particle in the granular space-time geometry. This property was known for the case the Riemannian space-time geometry of Kaluza-Klein [6, 7]. Now this property is generalized on the case of arbitrary granular space-time geometry.

Note, that according to definition of dynamics (17) all vectors of the skeleton are transported along the chain in parallel with itself (translation), i.e. without a rotation. It means a stronger definition of a free particle, than that, which is used conventionally. Usually the rotating particle, moving in the absence of external fields, is considered to be free, although some parts of the particle move with acceleration, generated by the rotation. In the free motion, defined by the relation (17), all points of the skeleton move without an acceleration and all vectors of the skeleton do not rotate. The particle rotation appears as a special kind of motion with superlight speed (with space-like leading vector of the world chain). This property seems rather unexpected from conventional viewpoint [9, 10, 8]. However, this property may take place in some special form of the granular geometry. Then the composite particle rotation is realized in the helical shape of the world chain.

It is quite reasonable, that the dynamic equations in the granular space-time geometry cannot be differential equations. The dynamic equations can be only difference equations.

Let the elementary length $\lambda_0$ have the form

$$\lambda_0^2 = \frac{\hbar}{2bc}$$

where $\hbar$ is the quantum constant, $c$ is the speed of the light and $b$ is the universal constant, defined by (15). Let the constant $\sigma_0$ in (14) be
small enough. Then the motion of a pointlike particle in the granular space-time geometry (14) appears to be multivariant (stochastic). Statistical description of this multivariant particle motion coincides with the quantum description in terms of the Schrödinger equation [4]. Quantum constant appears in the description via elementary length (18), which is a parameter of the granular space-time geometry.

2 Difference dynamic equations as a generalization of differential dynamic equations for pointlike particle in the Riemannian space-time

Let us show that dynamic difference equations (17) for the pointlike particle, described by the vector \( \mathbf{P}_s \mathbf{P}_{s+1} \) can be transformed to dynamic equations for a geodesic, if the space-time geometry is Riemannian. In the Kaluza-Klein space-time geometry, which is a 5-dimensional Riemannian geometry, the geodesic describes the pointlike charged particle motion in the given gravitational and electromagnetic fields.

At first, we consider the case of the Riemannian space-time without external fields, i.e. the pseudo-Euclidean space of index 1. For the pointlike particle the dynamic equations (17) take the form

\[
(\mathbf{P}_s \mathbf{P}_{s+1} \cdot \mathbf{P}_{s+1} \mathbf{P}_{s+2}) = |\mathbf{P}_s \mathbf{P}_{s+1}| \cdot |\mathbf{P}_{s+1} \mathbf{P}_{s+2}|, \quad s = \ldots 0, 1, \ldots
\]

\[
|\mathbf{P}_s \mathbf{P}_{s+1}|^2 = |\mathbf{P}_{s+1} \mathbf{P}_{s+2}|^2, \quad s = \ldots 0, 1, \ldots
\]

(19) (20)

Using relation (5) for the scalar product in the relation (19), one obtains from (20), (19) for the case \( s = 0 \)

\[
\sigma (P_0, P_1) = \sigma (P_1, P_2), \quad \sigma (P_0, P_2) = 4\sigma (P_0, P_1)
\]

(21)

Let us choose the coordinate system in such a way, that

\[
P_0 = \{0, 0, \ldots 0\}, \quad P_1 = \{x^0, x^1, \ldots, x^n\}
\]

\[
P_2 = \{2x^0 + \alpha^0, 2x^1 + \alpha^1, \ldots, 2x^n + \alpha^n\}
\]

(22)

\[
\mathbf{P}_0 \mathbf{P}_1 = \{x^0, x^1, \ldots, x^n\}, \quad \mathbf{P}_1 \mathbf{P}_2 = \{x^0 + \alpha^0, x^1 + \alpha^1, \ldots, x^n + \alpha^n\}
\]

(23)
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\[ \mathbf{P}_0 \mathbf{P}_2 = \{2x^0 + \alpha^0, 2x^1 + \alpha^1, \ldots, 2x^n + \alpha^n\} \quad (24) \]

Let us introduce designations

\[ x = \{x^0, \mathbf{x}\} = \{x^0, x^1, \ldots, x^n\}, \quad \alpha = \{\alpha^0, \alpha\} = \{\alpha^0, \alpha^1, \ldots, \alpha^n\} \quad (25) \]

where \( x \) and \( \alpha \) are \((n+1)\)-vectors, whereas \( \mathbf{x} \) and \( \alpha \) are \(n\)-vectors.

The equations (21) are reduced to the relations

\[ (x.\alpha) = x^0 \alpha^0 - x \alpha \equiv x^0 \alpha^0 - \sum_{\mu=1}^{\mu=n} \lim_{\mu=1}^{\mu=n} x^\mu \alpha^\mu = 0 \quad (26) \]

\[ (\alpha.\alpha) = \alpha^0 \alpha^0 - \mathbf{x} \alpha \equiv \alpha^0 \alpha^0 - \sum_{\mu=1}^{\mu=n} \lim_{\mu=1}^{\mu=n} \alpha^\mu \alpha^\mu = 0 \quad (27) \]

If the vector \( \mathbf{P}_0 \mathbf{P}_1 \) is timelike (\(|\mathbf{P}_0 \mathbf{P}_1|^2 > 0\)), then there is the unique solution \( \alpha = \{\alpha^0, \alpha\} = 0 \) of equations (26), (27), and the points \( P_0, P_1, P_2 \) lie on the same timelike straight (geodesic). If the vector \( \mathbf{P}_0 \mathbf{P}_1 \) is null (\(|\mathbf{P}_0 \mathbf{P}_1|^2 = 0\)), then \( \alpha = k \mathbf{x} = \{k\alpha^0, k\mathbf{x}\} \), where \( k \) is arbitrary real number, and the points \( P_0, P_1, P_2 \) lie on the same null straight (geodesic). If the vector \( \mathbf{P}_0 \mathbf{P}_1 \) is spacelike (\(|\mathbf{P}_0 \mathbf{P}_1|^2 < 0\)), the solution is not unique. It has the form

\[ \alpha = \{a, \mathbf{a} n\}, \quad n^2 = 1, \quad n \mathbf{x} = -x^0 \quad (28) \]

where \( a \) is an arbitrary real number, \( \mathbf{n} \) is a unite \(n\)-vector. The points \( P_0, P_1, P_2 \) lie on the same spacelike straight (geodesic), only if \( a = 0 \).

In general case \( a \neq 0 \), the spacelike vector \( \mathbf{P}_1 \mathbf{P}_2 \) is multivariant, if the vector \( \mathbf{P}_0 \mathbf{P}_1 \) is spacelike. As far as the quantity \( a \) may be infinitely large, the world chain with the spacelike leading vector \( \mathbf{P}_s \mathbf{P}_{s+1} \) appears to be impossible. This fact is known. At the conventional approach it is postulated. In the dynamic difference equations (17) the impossibility of the spacelike leading vector is a corollary of dynamic equations.

Let us consider the case, when the space-time geometry is the pseudo-Riemannian geometry of index 1. Then supposition, that the vector \( \mathbf{P}_0 \mathbf{P}_1 \) is timelike, and the points \( P_0, P_1, P_2 \) lie on the same timelike geodesic, is compatible with equations (21). According to (21) we have

\[ \rho (P_0, P_1) + \rho (P_1, P_2) = \rho (P_0, P_2), \quad \rho (P_1, P_2) = \sqrt{2\sigma (P_1, P_2)} \quad (29) \]
The timelike geodesic in the pseudo-Riemannian space of index 1 is the longest line. In other words, in the pseudo-Riemannian space of index 1 for any three points \( P_0, P_1, P_2 \), divided by timelike intervals "the triangle axiom" takes place

\[
\rho(P_0, P_1) + \rho(P_1, P_2) \leq \rho(P_0, P_2)
\]  

(30)

If one supposes, that the point \( P_1 \) does not belong to the geodesic \( \mathcal{L}_{P_0P_2} \), passing through points \( P_0 \) and \( P_2 \), then

\[
P_1 \notin \mathcal{L}_{P_0P_2} : \quad \rho(P_0, P_1) + \rho(P_1, P_2) < \rho(P_0, P_2)
\]  

(31)

Condition (31) is not compatible with dynamic equations (21) for timelike vector \( P_0P_2 \). Hence, for \( |P_0P_2|^2 > 0 \) the points \( P_0, P_1, P_2 \) lie on the same timelike geodesic. This statement is valid for any three points \( P_s, P_{s+1}, P_{s+2}, s = ...0,1,... \)

Thus, if the space-time is the pseudo-Riemannian space of index 1, the dynamic equations (17) for pointlike particle describe timelike geodesics of the space-time, if the leading vector \( P_sP_{s+1} \) is timelike. On the other side, timelike geodesics in the 5-dimensional Kaluza-Klein space-time describe motion of charged pointlike particle in the given gravitational and electromagnetic fields. Thus, the dynamic equations (17) are a generalization of conventional relativistic dynamics on the granular space-time geometry.

### 3 Composite particles

If the skeleton \( \mathcal{P}_1 = \{P_0, P_1\} \) consists of two points, it describes a pointlike particle. If the skeleton \( \mathcal{P}_n = \{P_0, P_1, ...n\} \) consists of more, than two points \( (n \geq 2) \) it describes a composite particle. Let the space-time dimension be \( N \). Then the number of coordinates, describing evolution of the skeleton \( \mathcal{P}_n = \{P_0, P_1, ...n\} \), is equal to \( nN \), whereas the number of dynamic equations (17) describing the skeleton evolution is equal to \( n(n+1) \). If complexity \( n \) of the particle composition increases, the number \( n(n+1) \) of dynamic equations increases faster, than the number of dependent dynamic variables \( nN \). At \( n \geq N \) the number of dynamic equations becomes larger, than the number of dependent variables.

For pointlike particle \( (n = 1) \) the number of dynamic equations is equal to two, whereas the number of dynamic variables is equal
to five in the 5D Kaluza-Klein space-time geometry, and it is equal to four in the 4D space-time geometry of Minkowski. In both cases the impossibility of spacelike world chain is conditioned by the fact, that the number of dynamic variables is larger, than the number of dynamic equations. One should expect, that for sufficiently complex particles with sufficiently large $n$, the world chains with spacelike leading vector $P_0^{(s)}P_1^{(s)}$ appear to be possible. In order that the world chain with the spacelike leading vector $P_0^{(s)}P_1^{(s)}$ be observable, the world chain is to have the shape of a helix with timelike axis.

Motion of elementary particles, which are not pointlike, is not yet investigated properly. There is only some information on the Dirac particle, whose skeleton consists of three points ($n = 2$), and the leading vector is spacelike [8]. In this case the world chain is a spacelike helix with the timelike axis. Such a spacelike helix cannot exist in the granular geometry (13). However, if the world function (13) is modified slightly at small distances $\sigma_{g} \rightarrow \sigma_{gm}$

$$\sigma_{gm} = \sigma_{M} + \lambda_{0}^{2} \left\{ \begin{array}{ll} \text{sgn} (\sigma_{M}) & \text{if } |\sigma_{M}| > \sigma_{0} \\ \left( \frac{\sigma_{M}}{\sigma_{0}} \right)^{3} & \text{if } |\sigma_{M}| \leq \sigma_{0} \end{array} \right., \quad \lambda_{0}^{2}, \sigma_{0} = \text{const} \geq 0 \quad (32)$$

such a spacelike helix becomes possible. The spacelike helix is possible also for other space-time geometries, where the the world function in interval $(-\sigma_{0}, \sigma_{0})$ has the form $f (\sigma_{M}/\sigma_{0}), |\sigma_{M}| < |\sigma_{0}|$. It is to satisfy the condition $|f (\sigma_{M}/\sigma_{0})| < |\sigma_{M}/\sigma_{0}|$.

Identification of the elementary particle with the helical world chain and the Dirac particle is founded on the following fact. In the classical limit the Dirac equation for a free particle describes a classical dynamic system having 10 degrees of freedom. Solution of dynamic equations leads to a helical world line with the timelike axis [9]. It is not quite clear, whether this helix is spacelike, or timelike, because the internal degrees of freedom, responsible for circular motion, are described nonrelativistically (i.e. incorrectly), although external degrees of freedom are described relativistically [10].

Note, that dynamic equations (17) could not describe the skeleton rotation directly. They describe only the parallel transport of all vectors $P_{i}P_{k}, i, k = 0, 1, \ldots n$. The phenomenon of rotation appears only in the form of helical world chain, and this circumstance generates restricted values of spin, which is supposed to be connected with the
Supposition, that the elementary particle structure is determined by mutual disposition of points in its skeleton seems to be natural and reasonable. Such parameters of the elementary particle as mass and electric charge are geometrized already for pointlike particle. In the case of pointlike particle its mass $m$ is defined by the relation (15), whereas the electric charge is determined by the projection of the particle momentum onto the direction, chosen by the space-time compactification of the Kaluza-Klein space-time. One should expect, that in the case of composite particles all parameters of the particle, including the mass and the electric charge, will be determined by disposition of points in the particle skeleton. Such an idea of complete geometrization of the elementary particles, when dynamics and parameters of the elementary particle are determined completely by the particle skeleton, seems to be attractive. At such an approach the particle skeleton is the only characteristic of the elementary particle. Such a description does not contain wave functions, branes, strings and other exotic matters, which are very far from the space-time geometry. The observed symmetries of elementary particles may be interpreted as symmetries of the points inside the particle skeleton.

4 Multivariance and discrimination

The granular geometry has two important properties, which absent in the conventional description of the space-time as a Riemannian space. The multivariance is conditioned by the fact, that the equivalence relation (7) has many solutions. Multivariance of timelike vectors is introduced in granular geometries (13) and (32) by the term with coefficient $\lambda_0^2$, where $\lambda_0$ is some elementary length. Multivariance of timelike vectors is responsible for quantum effects. Multivariance of spacelike vectors does not depend on $\lambda_0$. It is connected by the fact that the space-time geometry is close to the pseudo-Riemannian geometry of index 1.

Discrimination (or zero-variance) is conditioned by the fact, that the equivalence relation (7) has no solutions. The discrimination effect is responsible for discrete values of the elementary particle parameters. The most clear manifestation of this effect appears at the compactification of the Kaluza-Klein geometry. Compactification
means that the coordinate $x^5$, responsible for the electric charge, can change inside some finite interval $x^5 \in (-L, L]$, and the end points of this interval are identical. It means that all physical quantities (and wave functions) are periodic function of $x^5$ with the period $2L$. Compactification of the Kaluza-Klein geometry means a modification of its topology. However, in the physical geometry the topology is determined completely by the world function. One cannot change the topology independently of a corresponding change of the world function.

At the conventional quantum approach the eigenvalues of the momentum operator $p_5 = -i\hbar \partial / \partial x^5$ are $(\hbar / 2L)$-fold, and there are many geodesics, connecting any two points of the space-time. The electric charge appears to be multiple to some elementary charge, and this property is conditioned by quantum principles. At the conventional approach the world function is defined as a derivative quantity

$$\sigma(P, Q) = \frac{1}{2} \left( \lim_{\mathcal{L}_{PQ}} \sqrt{g_{ik}(x) \, dx^i \, dx^k} \right)^2$$  \hspace{1cm} (33)

where $\mathcal{L}_{PQ}$ is a geodesic, connecting the points $P$ and $Q$. As far as there are many geodesics, connecting two points $P$ and $Q$, the world function $\sigma(P, Q)$ appears to be multivalued. At the conventional approach, where the world function is a derivative (not fundamental) quantity, it may be multivalued.

In the physical geometry, where the world function is a fundamental quantity, it must be single-valued. If one defines the world function using the relation (33), it is necessary to use only one geodesic, removing another ones. Using the shortest geodesic, one obtains the single-valued world function. The situation looks as follows. One modifies the world function, and compactification is a corollary of this modification. Dynamic equations (17) for a pointlike particle impose restrictions on the electric charge of the pointlike particle [11]. These constraints have nothing to do with the quantum principles. They are purely geometrical constraints, conditioned by zero-variance of the space-time geometry compactification.
5 Transformation of space-time geometry to a standard geometry by means of introduction of geometric force fields.

The difference dynamic equations (17) can be written in the form, which is close to the conventional description in the Kaluza-Klein space-time. Let \( \sigma_{K_0} \) be the world function in the space-time geometry \( G_{K_0} \). The geometry \( G_{K_0} \) is the 5D pseudo-Euclidean geometry of the index 1 with the compactified coordinate \( x^5 \). In other words, the space-time geometry \( G_{K_0} \) is the Kaluza-Klein geometry with vanishing gravitational and electromagnetic fields. Let us represent the world function \( \sigma \) of the space-time geometry \( G \) in the form

\[
\sigma(P, Q) = \sigma_{K_0}(P, Q) + d(P, Q)
\]

where the function \( d \) describes the difference between the true world function \( \sigma \) of the real space-time geometry and the world function \( \sigma_{K_0} \) of the standard geometry \( G_{K_0} \), where the description will be produced. Then one obtains

\[
\left( P_0 P_1 Q_0 Q_1 \right) = \left( P_0 P_1 Q_0 Q_1 \right)_{K_0} + d(P_0, Q_1) + d(P_1, Q_0) + d(P_0, Q_0) - d(P_1, Q_1)
\]

\[
|P_0 P_1|^2 = |P_0 P_1|_{K_0}^2 + 2d(P_0, P_1)
\]

where index "\( K_0 \)" means, that the corresponding quantities are calculated in the geometry \( G_{K_0} \) by means of the world function \( \sigma_{K_0} \).

By means of (35), (36) the dynamic equations (17) can be written in the form

\[
\left( P_i^{(s)} P_k^{(s)} P_i^{(s+1)} P_k^{(s+1)} \right)_{K_0} - \left| P_i^{(s)} P_k^{(s)} \right|_{K_0}^2 = w \left( P_i^{(s)}, P_k^{(s)}, P_i^{(s+1)}, P_k^{(s+1)} \right), \quad i, k = 0, 1, \ldots n
\]

\[
\left| P_i^{(s+1)} P_k^{(s+1)} \right|_{K_0}^2 - \left| P_i^{(s)} P_k^{(s)} \right|_{K_0}^2 = 2d \left( P_i^{(s)}, P_k^{(s)} \right) + 2d \left( P_i^{(s+1)}, P_k^{(s+1)} \right),
\]

\[
i, k = 0, 1, \ldots n
\]
where
\[ w \left( P_i^{(s)}, P_k^{(s)}, P_i^{(s+1)}, P_k^{(s+1)} \right) = 2d \left( P_i^{(s)}, P_k^{(s)} \right) - d \left( P_i^{(s)}, P_k^{(s+1)} \right) - d \left( P_k^{(s)}, P_i^{(s+1)} \right) + d \left( P_i^{(s)}, P_i^{(s+1)} \right) - d \left( P_k^{(s)}, P_k^{(s+1)} \right) \] (39)

Equations (37), (38) are dynamic difference equations, written in the geometry \( G_{K_0} \). Rhs of these equations can be interpreted as some geometric force fields, generated by the fact that the space-time geometry \( G \) is described in terms of some standard geometry \( G_{K_0} \). These force fields describe deflection of the granular geometry from the Kaluza-Klein one. Such a possibility is used at the description of the gravitational field, which can be described as generated by the curvature of the curved space-time, or as a gravitational field in the space-time geometry of Minkowski. In dynamic equations (37), (38) such a possibility is realized for arbitrary granular space-time geometry.

Evolution of the leading vector \( P_0^{(s)} P_1^{(s)} \) is of most interest. These equations are obtained from equations (37), (38) at \( i = 0, k = 1 \). One obtains from equations (37), (38)
\[
\left| P_0^{(s+1)} P_1^{(s+1)} \right|_{K_0}^2 - \left| P_0^{(s)} P_1^{(s)} \right|_{K_0}^2 = 2d \left( P_0^{(s)}, P_1^{(s)} \right) - 2d \left( P_1^{(s)}, P_1^{(s+1)} \right) \]
(40)

\[
\left( P_0^{(s)} P_1^{(s)} P_0^{(s+1)} P_1^{(s+1)} \right)_{K_0} - \left| P_0^{(s)} P_1^{(s)} \right|_{K_0}^2 = 3d \left( P_0^{(s)}, P_1^{(s)} \right) - d \left( P_0^{(s)}, P_1^{(s+1)} \right) + d \left( P_1^{(s)}, P_1^{(s+1)} \right) \]
(41)

where one uses, that \( P_1^{(s)} = P_0^{(s+1)} \).

In the case, when the space-time is uniform, and the function
\[ d \left( P, Q \right) = D \left( \sigma_{K_0} \left( P, Q \right) \right) \]
(42)

the equations (40), (41) take the form
\[
\left| P_0^{(s+1)} P_1^{(s+1)} \right|_{K_0}^2 - \left| P_0^{(s)} P_1^{(s)} \right|_{K_0}^2 = 0 \]
(43)
In the case, when the leading vector $P_0^{(s)} P_1^{(s)}$ is timelike, one can introduce the angle $\phi_{01}^{(s)}$ between the vectors $P_0^{(s)} P_1^{(s)}$ and $P_0^{(s+1)} P_1^{(s+1)}$ in the standard geometry $G_{K_0}$. By means of (43) it is defined by the relation

$$\cosh \phi_{01}^{(s)} = \frac{\left( P_0^{(s)} P_1^{(s)} P_0^{(s+1)} P_1^{(s+1)} \right)_{K_0}}{\left| P_0^{(s)} P_1^{(s)} \right|_{K_0}^2}$$

Then in the uniform geometry $G$ the equation (44) has the form

$$\sinh \frac{\phi_{01}^{(s)}}{2} = \frac{\sqrt{4d \left( P_0^{(s)} P_1^{(s)} \right) - d \left( P_0^{(s)} P_1^{(s+1)} \right)}}{\sqrt{2} \left| P_0^{(s)} P_1^{(s)} \right|_{K_0}}$$

Thus, relativistic dynamics of particles can be generalized on the case of the granular space-time geometry.

6 Concluding remarks

The presented conception is completely orthodox, because it does not use any new principles. Introducing into consideration nonaxiomatizable geometries, one removes only incompleteness in description of the space-time geometry. Orthodoxy of the conception evidences in behalf of this conception.

In general, the geometric dynamics (17) is a classical dynamics in the granular space-time geometry. The granularity of the space-time generates two new properties, which are absent in the axiomatizable geometries: (1) multivariance, which is responsible for quantum properties, (2) zero-variance (discrimination mechanism), which is responsible for discreteness of the elementary particles parameters. The multivariance of the granular space-time geometry can be taken into account by means of the statistical description. Quantum theory can
imitate multivariance (and statistical description) on the level of dynamics, but it cannot imitate the zero-variance (discrimination mechanism). As a result the contemporary theory of elementary particles has no key to explanation of discrete parameters of the elementary particles.

Let us note in conclusion that we did not use any new hypotheses. Our conception is not a conceptually new theory. It is simply a generalization of the classical relativistic dynamics onto the case of granular space-time geometry, which was ignored by contemporary mathematicians (and physicists). Using granular space-time, we do not use any new hypotheses or principles. We have overcame simply the preconception, that the space-time geometry may be only axiomatizable. Besides, we reduce the number of principles in the theory in the sense, that the quantum principles are not used. Quantum effects are described now by multivariance of the granular space-time geometry. Impossibility of the spacelike world chain appears to be only a corollary of the fact, that such a world chain cannot be observed, but not a matter of principle.

The generalization of classical physics on the case of the granular space-time geometry is not yet accomplished in the sense, that only generalization of dynamic equations for the particle motion in the given external fields has been obtained. Another part of the classical physics, which describes influence of the matter on the space-time geometry (gravitation equations and Maxwell equations) has not been generalized yet on the case of the granular space-time geometry.

References


CONCEPTS OF INERTIAL, GRAVITATIONAL AND ELEMENTARY PARTICLE MASSES

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Abstract

In this article the concept of mass is analyzed based on the special and general relativity theories and particle (quantum) physics. The mass of a particle \( m = E_0/c^2 \) is determined by the minimum (rest) energy, which is necessary to create that particle and which is invariant under Lorentz transformations. The mass of a bound particle in any field is described by \( m < \)
$E_0/c^2$ and for free particles in the non-relativistic case the relation $m = E/c^2$ is valid. This relation is not correct in general for particles, and it is wrong to apply it to fields if we adopt the mass determination in particle physics. In atoms or nuclei (i.e. if the energies are quantized) the mass of the particle changes discretely. In non-relativistic cases, mass can be considered as a measure of gravitation and inertia similar to the $E/c^2$ in the relativistic case.
1 Introduction

Lev Okun published an article "on the concept of mass in the special relativity" [1] to show that the formula

$$E_0 = mc^2$$  \hspace{1cm} (1)

is correct but the formulas

$$E = mc^2,$$  \hspace{1cm} (2)

$$E_0 = m_0c^2,$$  \hspace{1cm} (3)

$$E = m_0c^2$$  \hspace{1cm} (4)

and

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$  \hspace{1cm} (5)

are wrong. In fact, Eqs. (2-5) are historical artifacts but, at present, they are still seen in many textbooks. Mass is invariant under Lorentz coordinate transformations. In reality, a lot of physicists take a position against Okun and the other particle physicists. For example, among the physicists working on general relativity Khrapko thinks differently than Okun and other particle physicists [2]. However, we cannot directly measure masses of particles with small and large velocities and comprise them. For small velocities, masses of elementary particles are estimated from the measured ratio $e/m$, which is given as masses of particles, because the value of electric charge is known. But in relativistic cases we may measure only impulses and energies of particles and the estimated masses depend on the relation between this quantities and masses. Therefore, the second method may give mass which is determined as $E/c^2$ and also is invariant under the Lorentz transformations or rest mass $E_0/c^2$.

The aim of this article is to discuss the concept of mass based on same fundamental principles of theoretical physics and on considering general theory of relativity. Also to show that Eq. (2) is correct in some cases and to show that the mass concepts, like the inertial and gravitational masses, are often misunderstood.

The oldest concept of mass is the one that was developed when people started trading and that was used later on as matter content.
in body. The concepts of inertial and gravitational masses were used in conjunction with Newton’s mechanics for the first time. Any physical quantity must be measurable in some way directly or indirectly. This perspective was concretized by Einstein when he constructed the Theory of Relativity. We can measure mass by using the laws of dynamics or with the scale. Therefore, matter content is nothing but the gravitational mass. In reality for the objects (not particles) a concept like matter content is not an exact thing and its measurement method is not different than that of gravitational mass. Therefore, we do not deal with it in this article.

2 New concepts introduced by particle physics and special relativity: Energy, momentum and the problematic mass

The least action principle is one of the fundamental principles in physics and action does not give absolute value of physical quantity, but shows only how processes proceed. But all physical processes in different inertial coordinate systems proceed similarly. Therefore, in order to obtain the most general results, we assume that action stays constant under Lorentz transformations. By multiplying invariant quantity by a dimensional constant we can change its dimension to the dimension of the action. We already have a quantity that is invariant under Lorentz transformations, namely, the interval between events, \( ds \) [3]. For the free particle in coordinate system where they are located the interval is given as

\[
\frac{ds}{c} = \sqrt{1 - \frac{v^2}{c^2}}\ dt \quad (6)
\]

Multiplying Eq. (6) by a dimensional constant (say \( b \)) and integrating we obtain the action as

\[
S = -\int_{t_1}^{t_2} b\sqrt{1 - \frac{v^2}{c^2}}\ dt \quad (7)
\]

Here, the minus sign is there to guarantee that the extreme point of the integral is a minimum, instead of a maximum.

The Lagrangian is obtained by taking the time derivative of the action. In the Langrangian functional,
the fact that all the terms except the term $b$ are taken from special relativity, makes the Langrangian valid for speeds close to the speed of light (and evidently for small velocities). Here, we consider a particle that is not under any external influence. Moreover, except its mass, such a particle has no any physical property that is important for the problem under consideration. Therefore, $b$ should be related to the mass (one can realize that since $b$ is constant, we have taken mass as a constant, automatically). For a particle moving with small velocity, expanding the Langrangian in the ratio $v/c$ and discarding the small terms, we obtain

$$L = -bc\sqrt{1 - \frac{v^2}{c^2}} \approx -bc + \frac{bv^2}{2c}.$$  \hspace{1cm} (9)

The constant terms of the Langrangian do not characterize the kinetics of the particle and can be omitted. Thus, to find the relation of the quantity $b$ with the mass of the particle we should look at the second term in the above expansion. The Langrangian for a free particle with small velocity is equal to its kinetic energy. Therefore,

$$\frac{bv^2}{2c} = \frac{mv^2}{2}.$$ \hspace{1cm} (10)

This leads to $b = mc$. Then the relativistic Langrangian is

$$L = -mc^2\sqrt{1 - \frac{v^2}{c^2}}.$$ \hspace{1cm} (11)

Most generally, momentum of a particle is the partial derivative of the Langrangian with respect to its velocity

$$\mathbf{p} = \frac{\partial L}{\partial \mathbf{v}}.$$ \hspace{1cm} (12)

and to find the components of the momentum vector the partial derivative of the Lagrangian with respect to each component of the velocity vector must be calculated. Now using Eq. (11) and Eq. (12), we find the momentum of the free particle:
\[ p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}. \]  

(13)

Naturally, for small velocities, \( v << c \) this equation becomes the well known Newtonian formula

\[ p = mv. \]  

(14)

In the above description that is based on special relativity the particle that is motionless or moving with small velocities has mass as seen from Eq. (14). There is no need to call mass \( m_0 \) in the nonrelativistic case because we treated mass as a constant. With this approach the coefficient \( \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \) has no physical relation with mass. In such an approach the equation that is seen in many textbooks

\[ m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]  

(15)

must be wrong because it contains the term of rest mass. This mistake comes from some historical artifacts, which are outside the scope of particle physics and Einstein’s theory of special relativity. The quantity mass in the Eqs. (11) and (13) is invariant under Lorentz transformations.

Eq. (13) is derived using the most important and trustworthy quantities and concepts of mechanics. This formula is valid for all speeds and approaches infinity as the speed gets closer and closer to the speed of light. Why for 100 years (since the work of Lorentz in 1899) rest and relative mass concepts are often used? Is the mass of a particle, like the electric charge, baryon and lepton numbers, an invariant quantity?

We can go back to the force concept that is known since Newton. Force is the derivative of the momentum with respect to time. The direction of the velocity of a particle, and the direction of the force on it can be quiet different. We can look at two different cases for demonstration:

case 1: when the force and momentum are parallel

case 2: when they are perpendicular.
Concepts of inertial, gravitational and elementary particle masses

When the force and momentum of the particle are parallel, the magnitude of the velocity changes but the direction of the momentum does not change. Then the derivative of the Eq. (13) with respect to time gives

$$\frac{dP}{dt} = \frac{m}{(1 - \frac{v^2}{c^2})^{3/2}} \frac{dv}{dt}. \quad (16)$$

When the force and momentum are perpendicular, the magnitude of the velocity does not change but the direction of the particle changes continuously. Then the derivative of the momentum becomes

$$\frac{dp}{dt} = \frac{m}{(1 - \frac{v^2}{c^2})^{1/2}} \frac{dv}{dt}. \quad (17)$$

The left hand side of this formula, which describes the change of momentum with respect to time (Eqs. (16-17)), is the force and the right hand side includes acceleration of particle. In Newtonian physics, the ratios of the forces to the accelerations are masses. But, are these ratios

$$\frac{m}{(1 - \frac{v^2}{c^2})^{3/2}} \quad (18)$$

and

$$\frac{m}{(1 - \frac{v^2}{c^2})^{1/2}} \quad (19)$$

define the mass? No, Eqs. (18) and (19) are coefficients due to the relativistic motion that are not representing mass correctly. It is normal that for different situations coefficients in equations are different. However, it is not natural that the mass of the particle depends on the direction of the force acting on the particle. It is well known that these two definitions of mass were first introduced by Lorentz as longitudinal and transverse masses. Unlike Newtonian physics, here, mass is not the ratio of momentum and speed, because in relativistic and especially ultra-relativistic cases, inertia depends not only on the mass, but also on energy (for example kinetic energy). Without considering general relativity, we should not generalize the concept of mass in Newtonian physics.
Many authors use both the words "point particle" and "extended object" when dealing with similar problems in the frame of special relativity. In this section we only investigate the kinematics properties of free particles. In an extended object particles are not free.

Recall that Eq. (11) describes the Lagrangian of a relativistic free particle. From this equation it is seen that as the velocity of the particle approaches the speed of light, the magnitude of $L$ approaches zero and notice that $L$ is always negative. Since the energy of a free particle is always larger than zero, in Eq. (11), the term, which describes quantities related with the kinematics of the particle and which are positive should be increased by the addition of a new term. This new term in the Lagrangian must also be valid in the Newtonian domain. The energy can be written as

$$E = p \cdot v - L.$$  

If we insert the Lagrangian which is valid in Newtonian physics to Eq. (20), then we obtain

$$E = p \cdot v - \frac{mv^2}{2} = \frac{mv^2}{2}. (21)$$

This is the kinetic energy for the non-relativistic particle. The corresponding energy in relativistic case can be found by using the formulae (11) and (13)

$$E = \frac{mv^2}{(1 - \frac{v^2}{c^2})^{1/2}} + mc^2 (1 - \frac{v^2}{c^2})^{1/2} = \frac{mc^2}{(1 - \frac{v^2}{c^2})^{1/2}}. (22)$$

From this formula we can see that as the velocity of the free particle approaches the speed of light in vacuum, the energy becomes infinite. When the velocity of the free particle is zero, its energy is $mc^2$, not 0.

Now, to determine the relation between the energy and momentum of a particle in the frame of special relativity, we subtract the square of the Eq. (13) from the square of Eq. (22)

$$E^2 - p^2 c^2 = \frac{m^2 c^4 - m^2 v^2 c^2}{(1 - \frac{v^2}{c^2})} = m^2 c^4,$$

$$E^2 = p^2 c^2 + m^2 c^4,$$  

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and
\[ m^2c^4 = E^2 - p^2c^2. \] (24)

From these formulas, it is seen again that the energy of a motionless particle (i.e. rest energy) is
\[ E_0 = mc^2. \] (25)

Using the same formulas (Eqs. (13) and (22)) we find
\[ p = \frac{E}\sqrt{1 - \frac{v^2}{c^2}}. \] (26)

The particles having zero mass (photon, graviton), their speed in vacuum is \( c \), so
\[ P = \frac{E}{c^2}. \] (27)

Therefore, these objects even though they do not have mass, they have momentum and thus exert pressure when the number of particles is high enough (because pressure is statistical quantity). Momentum, pressure and the inertia are not only related to the particles (matter) with mass.

If we take \( E/c \) as the fourth component of the 4-momentum, in the Lorentz transformations all the necessary conditions for the invariant 4-momentum would be satisfied. As known, components of the 4-momentum in two different coordinate systems can be written as
\[
P_x = \frac{P'_x + \frac{v}{c^2} E'}{\sqrt{1 - \frac{v^2}{c^2}}},
\]
\[ P_y = P'_y, \]
\[ P_y = P'_y \] (28)

and
\[ E = \frac{E' + vP'_x}{\sqrt{1 - \frac{v^2}{c^2}}}. \] (29)

The Eq. (29) above is the simplified version of the fourth component of the 4-momentum.
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\[ \frac{E}{c} = \frac{E' c}{c} + \frac{v}{c} P' \sqrt{1 - \frac{v^2}{c^2}}. \]  

(30)

The invariant quantity in nature is the 4-dimensional momentum (or the energy-momentum tensor). Energy and momentum are not conserved separately. The conservation of momentum and conservation of energy separately is just an approximation and valid only for some of the cases within errors. We can compare Eq. (22) with the square of the interval between two events

\[ ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = c^2 dt^2 - dv^2. \]  

(31)

Here, we see that also \( ds^2 \) is the square of a magnitude of a 4-vector, just like the \( m^2 c^4 \) term in Eq. (24). As \( ds^2 \) is invariant under Lorentz transformations, \( m^2 c^4 \) is also an invariant. That means transforming from an inertial frame to any other one does not change the mass of particles. We see from this and the above that the mass of a particle is Lorentz invariant.

We can explain the problems with understanding the concept of mass up to this point as historical artifacts. In the past, even some of the brilliant and famous physicists, were unable to understand which of the two different versions of energy, that were introduced in 1900 by Poincare as Eq. (2) and in 1905 by Einstein as Eq. (1), is relevant. Moreover, the longitudinal and transverse masses that depend on the velocity of the particle (object) that Lorentz introduced in 1899 have been around. At those times longitudinal and transverse mass concepts were there. It seems, even Einstein could only cope with these difficulties about 10 years after 1905. We should not forget that at those times there were no precise experiments on these subjects. General theory of relativity showed that attraction (gravitation) and inertia are properties of energy. Later, quantum physics (particle and nuclear physics) showed that the physical mass is a different and deeper concept. However, in some of the scientific journals and books the mass that depends on speed remained. This is mostly because this confusion about mass does not affect the results of the calculations.
3 The mass of particles and objects defined like inertia and gravitation

For the free particles that are considered as elementary until 1960’s (we do not go down to the quark level here), the most important properties were their mass and rest energies \( E_0 = mc^2 \). The velocity of the massless particles (photon and graviton) in vacuum is \( c \). As the mass of elementary particles increases the number of properties that characterize them, like electric charge, lepton number, baryon number, isospin, strangeness, etc., the number of type of interactions they can have increase. (A simple example for this is that neutrinos have only lepton numbers but electrons that are much heavier than neutrinos, have lepton numbers and also electric charges.) Now the question is this: Is the mass of these particles (or rest energies) conserved in any situations?

As it is known, special theory of relativity describes the physical relation between space and time and determines the invariant physical quantities for the processes involving any type of motion. This theory is one of the fundamental theory of physics that is valid when the gravitational field is constant and its gradient is zero, where the space can be considered as flat. Therefore, to understand the concept of mass, we should consider general relativity that has a more general scope.

3.1 The effect of interaction of fields on nonrelativistic particles

Recall that in any interaction field (electrical, baryonic or gravitational) in which the particle is bound to the field \( E < 0 \), the mass of sub-atomic particles differs from their free states. Nuclear physics shows us that the free proton has a mass of 938, 2 MeV/c\(^2\) and four of them makes 3752, 8MeV/c\(^2\) The mass of the free neutron is 939, 6 MeV/c\(^2\) when four nucleons come together to form the nucleus of the Helium atom consisting of two protons and two neutrons, about 28 MeV of energy is released. Therefore, the mass of the nucleons in the nucleus becomes less than the free ones. Rainville et. al. [4] conducted a very precise and direct test of atomic mass conversion into photon energy and determined that the relation \( E = mc^2 \) is valid at
least to a level of 0.00004. The binding energy of each baryon in a neutron star in its gravitational field is much higher than the binding energies in nuclear interactions. In some of the high mass and dense neutron stars the binding energies can be about \(0.1mc^2\). In the gravitational field of black holes, the binding energy can exceed \(0.2mc^2\). Evidently, for each sub-atomic particle the binding energy in Earth’s, Sun’s or normal star’s gravitational fields is much less than the energy needed to form a nucleus.

The discussion above shows that the elementary particles loose some part of their mass in the attractive fields of the nucleus or stars. This shows that, in fact, mass is not a conserved quantity in general (unlike the charges of elementary particles). Contrary to the attractive forces, the repulsive forces between the baryons and the spin interactions of fermions increase the mass of the particles in the systems (Another example is that, if a photon tries to traverse a superconductor, it gains mass. Moreover, standard model of particle physics predicts that the fundamental particles obtain their mass through the interaction with the field).

Above, we have seen that the elementary particles loose more mass (rest energy) when becoming part of neutron stars and black holes. On the other hand, they gain considerable kinetic energy in rapidly rotating neutron stars (pulsars) or black holes. The newly born pulsar has a temperature about \(10^{11}K\) and it can spin about hundreds of times per second. Because of these, for such a pulsar, the sum of thermal and rotational energy losses can be \((0.05 - 0.1)Mc^2\) (i.e. 5-10 of sum of all the rest energies of all the particles constituting the pulsar). Pulsars loose energy in general by emitting neutrino pairs under the collapse and neutron star formation, magnetodipole radiation, ultrarelativistic and relativistic particles.

Naturally, electron-positron pair annihilation which occurs due to electromagnetic (and sometimes due to weak) interaction has high probability of occurrence in the reverse direction as well (pair creation). These types of interactions show that in the interaction of elementary particles, even all the mass of particles can transform into radiation and radiation can transform completely into mass.

Therefore, the experiments on Earth and observations of the processes in the Universe prove that the formula \(E = mc^2\) (for the bound particle) by Poincare in 1900 and the formula \(E_0 = mc^2\) (for the free...
particle) by Einstein in 1905 are both correct. This means that these formulae explain the partial mass-loss (by electroweak and gravitational interactions) or complete mass-loss (by electroweak interaction) to radiation. Here, if we go beyond the concept of the mass that is related to the rest-energy of the particle, by the changing of mass with interaction, we can see that we have returned to the Poincare formula. Because, always in nuclear reactions the emitted energy is calculated using the formula $\Delta E = \Delta mc^2$.

The experimental and observational proofs of a formula do not necessarily show that all types of description of that formula or the ideas behind the formula are always correct and true. In the nuclear and electromagnetic interactions part of the mass of the particles transforms into radiation (until now we could not observe the gravitational binding energy transforming directly into radiation) and if the mass-loss is denoted by $\Delta m$, the emitted energy is $\Delta E = \Delta mc^2$. However, if the energy of the particle increases by $\Delta E$ its mass in general does not increase. For the bound particle $\Delta m = \Delta E/c^2$ formula is always correct and for radiation $m = h\nu/c^2$ formula is never correct.

Poincare formula ($E = mc^2$) might be partially correct since the masses of particle having high Fermi energy or thermal energy in compact star can not exceed their rest mass. This formula is valid only when the particle is bound since the kinetic energy of the bound particle does not increase the mass of the particle.

3.2 Definition of inertial and gravitational masses

In nuclear physics, high-energy physics and astrophysics, the type of matter and mass as used in chemistry always changes. Sometimes matter changes into electromagnetic wave or the vice versa, it is created by electromagnetic wave. In all these processes the conserved quantity is neither the molecules or atoms themselves nor the elementary particles nor their masses. The conserved quantities are the different types of charge (electric charge, lepton number, baryon number).

In Einstein’s theory of general relativity, the equivalence of inertial and gravitational masses is a postulate. Thus, there is only one mass in general relativity theory. Inertial and gravitational masses are different manifestations of the same mass.
At a point, or in a very small local region of space-time, general relativity gives the same results as special relativity. In such a case since there is the possibility to have no gravitational field, the need for the gravitational mass disappears for the particle (body) in the theory of special relativity. Therefore, in special relativity only one mass term, the inertial mass term can be used. In accelerators, magnetospheres of pulsars, shock waves of Supernova remnants and in other cosmic objects, the things that are accelerated to the velocity close to the speed of light are the elementary particles not even the atoms (except some ions of atoms). These particles have masses specific to themselves and these are the same masses that particle physicists use \( m = \frac{E_0}{c^2} \) and which is the inertial mass of the particles. It is well known that the mass does not change and it is the same in all cases. Therefore, there is no need to an inertial mass term. As the energy of the particle (body) increases its inertia increases as well. Inertia is a measure of energy. Since a particle (body) has a rest energy it has rest inertia too. This inertia, in Newtonian physics, is defined as if it is a property of the mass.

Cosmology shows that the radiation and any type of field gravitate. The gravitation is not only a property of mass, but it is mainly a property of energy. Therefore, if we take special and general theories of relativity and particle physics as fundamental, we can say that mass is not the only source of gravitational field, or the only source of inertia. Mass, having specific properties, is the quantity that corresponds to the minimum energy for the creation of free elementary particles (for example \( \gamma + \gamma \rightarrow \text{rest energy of any particle pair} \)) or the quantity that corresponds to the rest energy \( (m = E_0/c^2) \). The energy of free particles can be changed continuously but their masses are discrete (for different types particles), well defined and specific to the particle. Only in this sense mass gains a different importance. When the energy of the particle (body) is close to its rest energy we are in the domain of Newtonian physics and we can consider the properties of energy as if they are properties of mass and this makes the calculations easier (we do not see \( c^2 \)'s wandering around).

4 Conclusions

Only free elementary particles have constant rest energies and corresponding masses \( (E_0/c^2 = m) \). The mass defined in this way
has a special meaning in quantum physics (particle and nuclear) and in all other physical branches. This mass is the ratio of the minimum value of energy needed for the creation of a particle to \( c^2 \). The value of minimum energy which is needed for the creation of a particle which is in a field and has negative energy is \( E < E_0 \). And in quantum states, this mass changes discretely. The particle in an atom that is isolated from external fields has a determined and constant mass.

It is more worthwhile and correct to consider the mass of the particle (body) as Lorentz invariant. In that case, the coefficient \((1 - \frac{v^2}{c^2})^{1/2}\) for the particles moving with relativistic speeds is not physically related to the mass of the particles as defined above. Then, in this case, the same coefficient (with different exponents) that enters the equation for the relation between the forces and accelerations has no relation to mass.

When the energy of the particle is close to its rest energy, Newtonian physics become applicable and therefore we can consider the properties of the rest energy as if they are properties of mass and this makes the calculations easier by getting rid of \( c^2 \). For all objects in Nature, when their heats and rotational or translational velocities change, at the same time their energies, inertia and attraction change, whatever their states (gas, liquid or solid) are. However, these changes are much less than the energy related to the total mass of all the particles of the systems and therefore for objects the Newtonian mass can be used. The gravitation properties and pressure of particles with the speeds of light depend on their energies. All types of energy (rotation, heat and the energy of fields) gravitate and in the most general sense have inertial properties.

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TEACHING
EXPLOITING THE INTERNET RESOURCES IN PHYSICS EDUCATION

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Abstract

The significance of the world wide web - Internet permanently increasing. The first computer network was prepared by physicists working in European Nuclear Research Centre - CERN. This article concerning possibilities using of the contemporary Internet sources at all stages of the physical education. These possibilities are selected in twenty three points and shortly discussed in main part of this article. In ending of this paper the merits and dem
Introduction

The Internet - a global system of interconnected computer networks - is by far one of the most revolutionary tools of our time and virtually has become a part of everyday life. It is also a tool that has revolutionized the process of teaching and learning like nothing before. It has transformed every room with a computer and an Internet connection into the vastest library and classroom one could ever imagine.

It is also an evident fact that in the new millennium, the World Wide Web - a system of interlinked hypertext documents accessed via the Internet - is an ingenious invention. It dates back to 1989 when physicists while working at European Nuclear Research Center - CERN, found the solution to the problem of merging the technologies of personal computers and computer networking into a powerful and easy to use global information system.

According to a recent own study, there have been more than 980 billion documents on the Web, including 270 million in Polish. (Results from march of 2009 year). By far the most Web content is in English. It is clear that the breathtaking growth of the Web is leading to information overload. Finding what you want is not easy, and becoming proficient at selecting sources requires relevant experience. Effective instruction is necessary to assist students in navigating the Internet.

In its widest sense, physics education refers to both the methods currently used to teach physics and to an area of pedagogical research that seeks to improve those methods at any level of education, middle and secondary grammar schools, colleges (baccalaureate programme) and universities with physics as an area of study as well as postgraduate studies and those leading to doctoral degree.

Moreover, physics education includes additional training for physics teachers as well as popularizing and disseminating knowledge of physics to a broad public - young people and adults. The target participants in physics education are schoolchildren, college and university students, schoolteachers, academic teachers, research workers, and adults within the general public.
Physics education online

Historically, physics has been taught at the secondary school and university level primarily by the lecture method, together with laboratory exercises aimed at verifying concepts taught in the lectures. Let us try to examine, in a systematized way, the new possibilities of exploiting the Internet resources in physics education.

1 Factual information

Internet resources can be made good use of alongside existing classic information resources such as textbooks, lecture notes, scholarly journals, scientific books for the general public, slides, CD-ROMs, films, and the mass media. A good example of websites containing physics-related factual information including news of achievements in physics are numerous sites linking to the Virtual Universe and Wikipedia [1-3]. Information on the websites can take many forms, including that of a crib [4].

2 Information about major events in physics

The Internet with its potentially unlimited resources opens a new world to teachers and other information seekers, and makes formerly out-of-reach information accessible. On websites there are texts about significant events in physics such as discoveries of new phenomena or objects and nominations to the Nobel Prize - widely regarded as a highly prestigious award, not only among physics [5]. Worth noting are the pages containing information of the most recent accomplishments in physics, still in a draft form waiting to be edited and published [6].

3 Physics quizzes, tests, and exams online

Successful attempts to utilize computers or so called electronic digital machines to assess students’ performance and achievements had already been made in the early 1960s. Modern computers have by far outclassed their ancestors in terms of technical and performance parameters. Scientifically objective physics tests on instructional websites, generally constructed by schoolteachers and academics [7,
8] constitute an excellent technique of instant evaluation of testees’ performance. A major plus point is that you get evaluated instantly. Sets of solved and unsolved physics problems designed for a wide range of students are ideal for physics test preparation. Reviewing and mastering these problems in physics help you enhance your problem solving skills, provide instant feedback on your progress, and prepare for college or university tests or exams.

4 Collections of physics problems

Physics problems constitute a crucial element in physics education. Hence, sets of problems that cover the key areas of physics are often published in a book form. Websites are a fit place for a variety of physics problems. These are both the problems selected from the collections of problems in a book form and the problems made ready to be posted on the Internet. The problems are put by teachers of physics and academic teachers and are aimed at school and university students [9]. A more detailed analysis shows yet another phenomenon of pupils and students requesting the solutions to the problems and/or encouraging the authors to give detailed guidance on solving relevant problems [10]. Physics problems can also be found on various discussion forums that are held on the Internet.

5 Olympiads and physics competitions

While surfing resources on the Web, looking for information on Physics Olympiads and physics contests it becomes evident that, as in the case of physics problems, they fall into two separate categories. One category comprises sites containing information on organization and entrance requirements the participants have to meet as well as the sets of tasks and problems. The creators of the sites are the institutions in charge of preparations for and staging Physics Olympiad or competition [11]. The leader among these is the Organizing Committee of the Polish Physics Olympiad, Regional Committees, and the Queen Jadwiga Fund of the Jagiellonian University which organizes competitions entitled ’wietlik’ [Skylight] [12]. The other more numerous list of sites contains views and opinions, commentaries, and occasionally the solutions to the Olympiad problems [13, 14]. There is a wide diversity of creators of these sites though schools and private persons seem to prevail.
6 Physics lessons scenarios for teachers

There is a wealth of useful material for teachers making preparations for their physics classes. These are, among other things, detailed descriptions of the procedures for the execution of the content of the course, with a focus on key uncertainties, as well as the formulations of lesson topics and syllabuses. Noteworthy are physics lessons scenarios [15] - outlines of the lessons giving particulars to detailed plans and possibilities. Such scenarios can be found on the sites that are administered mainly by schools or private individuals.

7 The descriptions of physics experiments as an extension of teaching materials

Experiment plays many roles in science, one of which is to test theories and to provide the basis for increasing cognizance of reality. Experiments online may have different levels of difficulty and the possibilities of performing them vary considerably in costs and sophistication - with experiments that require professional and costly apparatus and those which can be conducted with the use of some household items [16, 17]. For school students of importance is the former type of experiment which is likely to involve them and encourage them in performing on their own. Apart from the descriptions of online experiments we can also find users’ inquiries and requests for help with their homework [18].

8 Visualizations and demonstrations for physics teachers

Polish Internet resources concerning computational visualizations in physics are rather modest. Individual experiments with visualized demonstrations predominate on home pages administered by schools and private individuals. A fairly large number of pages is on the successive editions of the All-Poland Competition for the Demonstration of Visualization in Physics held by the Cracow Division of the Polish Physics Association in close collaboration with the Department of Physics and Applied Informatics at the School of Mining and Metallurgy, and the Institute of Physics, Jagiellonian University [19].
The resources in English, and particularly American English, which have been created by under the patronage of the PIRA - Physical Instructional Resource Association, are quite substantial. A number of American colleges and universities have participated in the project, including the University of Minesota [21]. There are numerous links to similar websites, for example, to recommended textbooks, which increases their usefulness [22].

9 Information on professional advancement for teachers

The Internet is a major source of information in most areas of life in any social activity. This also concerns professional advancement of physics teachers. Advancement requires the teacher to meet defined criteria related to teaching practice and professional development. Information on such matters as well as that concerning the requirements to be fulfilled can be found, generally, on the home pages of educational institutions that administer them. [23]. In addition to that there is a lot of professional counseling and legal advice with commentaries on relevant regulations and legal articles [24]. These are helpful to those who aspire to take the next step along their career path. There are also job offers for teachers and the addresses and telephones of educational institutions, and the users’ opinions about professional advancement.

10 Virtual reality simulations in the physics education

The recent development of a computer-based virtual reality simulation programme is a solution for realistic hands-on experimentation. Physical phenomena that are neither easy to perceive nor to measure in usual experiments can be presented in a virtual world and viewed in many different perspectives. Among some good examples of dangerous, high cost, and complicated experiments that have been realized in a VR system that are available on the Internet are: Brownian movement, Compton effect, and nuclear reaction. The system allows the user to display, move around, and interact with virtual worlds. He can change the viewpoint to see the elements of the
machinery, switch the direction or the intensity of the magnetic field, adjust the frequency of revolutions, etc, and influence the course of a virtual reality experiment. The creators of the sites with simulation experiments are students, teachers, research workers, and other information technology enthusiasts [26]. Apart from simulations, some pages contain commentaries and descriptions of physical phenomena.

11 Catalogues of teaching aids for teachers

The latest version of online catalogue designed to provide physics resources features thousands of items offered by renowned companies, such as Leybold or Phywe Systems - the largest manufacturer of educational scientific apparatus. Printed versions of catalogues are really voluminous and heavy, which means high distribution costs for relevant firms. Therefore, the manufacturers of educational teaching aids are looking for new and more effective means of informing potential clients. One such way is the Internet [27] - a source of a number of resources. It is a place where a full list of offers by a number of renowned manufacturers can be found. They also provide instructions for selected products and articles on the history of their first appearance as well as other topical issues [28, 29]

12 Physics popularization in academia

Today, most physics departments throughout the world have their own home pages available on the web site with information on an organizational structure, faculty, selected publications, course offerings, research trends, etc, as well as resource links and information about the various support networks [30, 31]. A substantial number of institutions promote physics with a range of activities, such as delivering public lectures on to a general audience, arranging workshops for students and teachers, and discussion panels. Information on such events can be found on the relevant department’s home pages.

13 Virtual reality field trips

Some research departments dealing with physics offer virtual reality trips on the Internet to those who wish to visit their laboratories.
Such trips are possible thanks to certain Web pages which provide information about research trends developed by the relevant department, appropriate route maps as well as numerous photos featuring apparatus along the route. A good example of the research departments which offer virtual trips are the laboratories dealing with ultrastrong magnetic field in Lolland and the US as well as the CERN - the European Organization for Nuclear Research [32, 33]. Without doubt, Web-based virtual reality field trips provide a more diverse and effective learning experience to students who may never have the opportunity to see such things in person.

14 Descriptions of physics laboratory apparatus and equipment

On the web pages we can find descriptions of equipment and apparatus used for physics experiments. They may vary in their dimensions and applications. Noteworthy is, for example, an extremely sensitive electronic electroscope, which consists of an electro-luminescent diode, an unipolar transistor, a resistor, an 9V battery [34]. A special mention deserve: a Marx generator, Tesla coil, high-voltage power supply, and a natural phenomenon called Jacob’s ladder [35]. These web pages are addressed both to young people and teachers. The descriptions of pieces of equipment, often very detailed, contain accounts of the problems encountered by their constructors. Apart from this, we can view discussion forums for those who have built a particular piece of equipment and who share their experience and problems that occurred during construction and operation..

15 Online physics lectures for school students and university students

In the Internet age, we potentially can use the power of the classroom teaching plus the Internet to produce a new type of global education, an open source model of training. Many academic teachers and researches across the world are putting their lectures on the Internet, either on a special website, such as school, community, blog sites or on an information site of the educational institution where the author is employed [36,37]. Lectures vary in quality and exactitude,
Exploiting the internet resources in physics education

starting from very exact, carefully elaborated diquisitions in the form of textbooks, to a set of handouts containing a list of issues to be discussed. Such lectures are mainly intended for university students but they may be easy enough to comprehend and follow for middle and secondary school students, and for use with homework and self-study assignments.

16 Physics education software

Both in the education of physics and in physics research there are endless possibilities for exploiting software, mainly for simulation and calculating. Furthermore, the Internet offers a vast number of web sites that provide varied educational software for use in physics teaching and research work [38]. The websites are controlled, operated and administered by research and academic institutions, schools and private persons [39]. They often offer free access downloads.

17 Paradoxes and curiosities

The paradoxes and curiosities of physics seem to be attractive element in the physics content area. Therefore, web authors readily create accessible web pages in the hope of enhancing users’ interest in them [40]. The same can be said of the use of paradoxes, optic/visual illusions and brain-twisters. In most cases such physical curiosities pages are administered by schools or individual persons [41, 42]. Regretfully, the authors of these web pages do not use paradoxes and curiosities to their full didactic potential as they fail to provide adequate explanations.

18 Students’ participation in online discussion forums

Participation in an online discussion group allows the user to actively engage in online communication with other participants. He is both a recipient and a creator/author of information. Forums or newsgroups are popular among many users who readily visit discussing places and give their opinion on the Internet. Unfortunately, the lack of censorship does not bring about the lack of self-censorship,
limited vocabulary, poor enunciation and offensive language are often a reflection of an author’s mind. Discussion topics may vary according to intended objectives, for example, a physics problem or an event in physics, such as an important discovery or a reception of the award for scholarly achievements. In some cases the participants’ statements is a clear evidence of not only their low intellectual standards but also linguistic ineptitude reflected in limited vocabulary, grammatical and spelling mistakes. Another issue involves the problem of verification, preservation, updating, and revising the statements. On some sites we can view statements which are two or more years old, on some of the others they have been deleted after few weeks.

19 User generated content complement in the printed media online

User generated content on the web is a phenomenon which has occurred only recently. Internet users generate their own content and distribute it on a variety of websites. Obviously traditional printed media are facing a new competitor in the media market: the user. Some books with complemented content are available on the Internate. Most of scholarly journals online, however, date back to a few years ago. The more recent ones have only a table of contents and some selected article. This is due to a rather laborous process of digitization of the older materials. Newer materials have generally been converted into a digital form. For example, Postpy Fizyki [Advancements in Physics] is available in both online version and printed version [45]. Some other periodicals post mainly general information which is informative in nature, for example, about the subject matter [46]. As for the online books, they may have chapters complemented with extended information and software for simulating some physical phenomena, such as diffusion and body melting.

20 Physics biographies

A computer with adequate software is an excellent tool for sorting out and searching according to the assigned criteria from among a vast array of elements. It is these types of operations that are performed while creating databases and biographies. That is why online
physics bibliographies prove so useful. Bibliographies of this type are created by information departments of scientific libraries, specially appointed for the purpose as well as by schools and individual persons [47]. In the nature of things, biographies compiled by the former group of above mentioned institutions are more extensive and professional in character. Biographies created by the latter group may come in useful as a physics teaching aid in a given type of school [48].

21 Organizational information for the members of physics associations

Physics associations of different countries, such as the Polish Physical Society or the American Physical Society have their web pages with information on organization structure, the board, membership, membership fees, policy, publications, programmes, etc. As a general rule, physics societies’ homepages are carefully prepared with contact information, and equipped with numerous links to www pages that are devoted to physics journals and physicists.

22 Video-films featuring experiments in physics

By way of the Internet we can gain access to short video-films that feature physical experiments [51, 52]. Longer films may require more memory and obviously much higher transmission speed. Therefore only short films are accessible on the Internet which last only no more than dozens of seconds to a few minutes. Such films show the most critical moments in the process of the relevant phenomenon. One of the advantages of such films is that they a show can be repeated several times, which allows users to observe the interesting moments in the experiment more closely.

23 Scholarly articles in the area of physics

Scholarly communication is undergoing transformation due to the rising costs of print and paper. The increase in subscription fees also presents a serious threat to library purchasing capability. A solution to the problem is the necessity of designing more effective electronic publishing systems, creating digital libraries, and more subscribing to...
online content. There are numerous databases which store numerous
texts on the relevant from different science disciplines, for example
mathematics and natural science [53, 54]. Subscribing to journals
online may mean lower costs and easier and faster accessibility to a
complete set of articles or other content by means of hypertext and
search engines.

The merits and demerits of the Internet

Taking everything into consideration, it is evident that the elec-
tronic resources can successfully supplement the educational process
at all levels of physical education by all its participants: school
and university students, schoolteachers, academic teachers and re-
searchers and other physics enthusiasts.

Making use of the Internet has its outstanding merits, for exam-
ple, easy and ready access to a wealth resources and any kind of
attractive, diverse, and up-to-date information on any topic under
the sun. Accessing the content on the relevant pages is fairly simple,
since the user only needs to know how to handle a search engine. And
the whole process takes no more than just a few tenths of a second.

Another good point about using the Internet is that, except for
broad band Internet fees users pay, the Internet is free of charge and
most Web access is anonymous. Only few pages require authentica-
tion and seven fewer require the user to pay a fee to access.

Yet another aspect of the Internet that is appealing to users is
that graphic design and layout ensure that content is presented in
an attractive style and easy to read or view. Important pages have
hyperlinks to them whose function is to direct a Web browser to
the address they link to. Appart from text and images, sound can be
used as a carrier of information. Some pages offer access to animation
and videofilms. All this makes up a hypertext that is displayed on a
computer, with references, or links to other text that the user can
immediately follow.

An important aspect of the Internet is that it enables the user
customization. Thanks to the hypertext the user may, for example,
choose conditions for displaying data, the size of the font, a display
format, etc. All those who are searching for the relevant information
for the assignments: learners beginning their physics course as well
as students and professors may make a good use of the Internet.
Exploiting the internet resources in physics education

As all man made systems and devices the Internet is not foolproof either and has it obvious drawbacks. One of which is the issue of the quality of information which is offered on web sites. Since anyone can post their texts on the Internet and to a large extent they are uncensored. Unfortunately the currently available search engines fail to check the validity of information. Although many sites offer high quality information there are reports of inaccurate and misleading facts on the Internet. As a result some young physics learners may have problems with assessing the validity of the relevant texts.

Yet another drawback is the fleeting/transient aspect of online information resulting from the accidental disturbances in accessibility to the Net. Each of the web pages can be disconnected at any time by the administrator. Taking this into consideration, users should remember of saving the relevant data as they may disappear for ever. To sum up, it is evident that the merits of the use of the Internet in physics education outweigh the dismerits, and against which we can protect ourselves.

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HEISENBERG’S UNCERTAINTY PRINCIPLE
IN HIGH SCHOOL CURRICULUM

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Abstract

The progress in teaching should be in accord with the progress of science, that’s why the subject of teaching must be supplemented by the elements of modern knowledge which permit pupils to understand better what happens around us. For better understanding microworld we must use Heisenberg’s uncertainty principle. The beginning of XXI century is characterised by intensification of microworld research, which we deal with in this paper.
"The more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa."

\[ \Delta p \Delta x \geq \hbar \]  

(1)

\( \Delta p \) - accuracy of momentum determination  
\( \Delta x \) - accuracy of position determination

From the analysis of physics handbooks and pupils’ statements we can say that the principle above is understood incorrectly by them. There are some reasons why is that (not penetrated in details): Namely:

- It is a difficult subject relating to the microworld and not available by pupils’ observation.
- Some teachers may have some problems with correct presentation of this subject.
- There are many different ways of described principles in handbooks which may cause chaos.
- Lack of clear requirements level in school curriculum and lack of principle used for describing its usage.
- General treatment of this subject by some teachers and handbook authors.

We can ask the question: should the uncertainty principle be in high school curriculum (profile classes)? The answer is affirmative (because of fast nanotechnology developing), but we should strike a happy medium, it means we should arrange a clear way of principle presentation, easily understood by pupils.

Based on curriculum and handbooks analysis we suggest the following track of reasoning.

**Description of uncertainty principle on the high school level.**

This description should have:
Heisenberg’s uncertainty principle in high school curriculum

1 Introduction,

which allows pupils to understand the microworld better. It should be linked with the macroworld for better comprehension by pupils and it ought to go to the microworld by analogy. The whole presentation should have examples.

The macroworld is everything occurring around us and we can see or touch it. That is why, the measurements which we take are easier than in the microworld. It happens like that because we can find out that our measurement does not influence on the state of the measured object in the macroworld.

2 Example

We are measuring the height of liquid column by the ruler. We are repeating this activity few times; even though we are getting different results, we know that it depends on inaccuracy of the measurement (instrument or observer). We are conscious that in the macroworld the measurable thing does not change its size after some time (in the same experimental conditions).

But it is not right in the microworld. In the microworld our intuition, which we obtained in the macroworld, starts to let us down. In some physicist’s opinion, this quantity has real meaning, which we can measure.

An example from the macroworld and the microworld.

The Moon circles around the Earth, which we can observe due to the sun light, which is reflected to our direction. The light in some way perturbs Moon’s movement, but the effect is insignificant.

We have a completely different situation when we think of electron’s motion. Even though we are observing some analogy between both configurations, the rules in the microworld are completely different. When we want to see the electron, like the Moon, we must reflect the light or another particle off it. In the electron case; the photon reflection does not leave the configuration without changes as it was in the macroworld. A photon, which will reflect from the electron, will modify electron’s motion and the change is not possible to avoid or to measure.
The fact, that we can’t in a classical way describe the electron’s motions, finds the reflection in Heisenberg uncertainty principle.

3 Subject development

leading out and conceptualization uncertainty principle.

Many interesting conclusions result in the wave theory. Inter alia that, there are no exist bodies with infinitely small sizes, and we cannot pinpoint the momentum of the particle (among others it results from Shannon formula - the fundamental formula of information theory).

From the fundamental formula of quantum theory

\[ p = \frac{\hbar}{\lambda} \]  

(2)

results, that if the value of momentum decreases, the wavelength will increase (the position of particle is fuzzy) and we cannot pinpoint the position of particle. Werner Heisenberg noticed this fact and he formulated uncertainty principle in 1927: ”The more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa.”

\[ \Delta p \Delta x \geq \hbar \]  

(3)

\( \Delta p \) - accuracy of momentum determination
\( \Delta x \) - accuracy of position determination

At first it was though that the measurement of the particle position introduces perturbations in its movement causing velocity and momentum changes. Today we know that uncertainty principle is not the mistake made during measurements. This principle shows some limitations where our classical imagination, for the description of this situation, does not exist.

This principle was confirmed many times by experiments. This is one of these experiments.
Heisenberg’s uncertainty principle in high school curriculum

We have the electron beam which moves with some velocity, and then we take one electron and we are trying to measure its position and velocity in x-direction. From the electron beam with momentum \( p \), which is parallel to 0y-axis, we separate electron beam about definite values of the coordinate x by means of the slot with the width \( \Delta x \).

![Figure 1: Deflection of electron beam on the single slot](image)

According to de Broglie’s hypothesis: electrons have some waves attributes, that is why we can get difraction image on the screen, which is situated opposite the slot and in long distance from it.

Opposite the slot appears a bright disk which is rounded by dark and bright circles alternately. The direction which is determined by the first minimum makes the angle \( \phi \) with orthogonal direction to the plane where is the slot. This angle fulfills the condition (analogically like for the light difraction):

\[
\Delta x \sin \phi = \lambda \tag{4}
\]

When we take the de Broglie’s formula \( \lambda = \frac{\hbar}{p} \) we are receiving:

\[
\Delta x p \sin \phi = \hbar \tag{5}
\]
Diffraction image develops on the screen, because the electrons are getting $\Delta p_x$ - element of momentum in the 0x direction, after going through the slot. As many as 96% of all electrons will be in the first maximum, that is why we can say, that the $\Delta p_x$ may have a value $p \sin \varphi$.

Taking (5) into account we get the following correlation:

$$\Delta x \Delta p = \hbar \quad (6)$$

When we also take the electrons, which are out of main maximum then (6) we can write:

$$\Delta x \Delta p \geq \hbar \quad (7)$$

The above correlation shows Heisenberg uncertainty principle. After the electrons go through the slot we receive the information about $x$ - coordinate with $\Delta x$ accuracy, but in return undetermined element of momentum comes into being. From this principle flows that: if we know the position of a moving particle, there is uncertainty in knowing its momentum, and vice versa. Uncertainty principle deals only with the nature of the microworld objects and it does not result from the inaccuracy of the instruments [1]. Measuring instruments enforces additional limitations on the precision of measured values pairs embraced with uncertainty principle (for pairs: $x$, $p$ or $E$, $t$ and so on).
4 Summary

Uncertainty principle explains us the nature the wave-particle duality of the light. It results from it that the two opposite aspects cannot appear in the same time and in the same experimental conditions. Therefore, if we check the wave character of the electron, the particle face will be invisible. In turn, during checking the particle character, the wave character will be invisible. Particles and photons we can compare with coins that we can only look at one side at one moment.

Comprehension questions:

- How does Heisenberg’s uncertainty principle affect the determination of a particle’s location and velocity?

- Where does Heisenberg’s uncertainty principle have practical importance?

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RELATIVISTIC FORCE TRANSFORMATION

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Abstract

Formulae relating one and the same force in two inertial frames of reference are derived directly from the Lorentz transformation of space and time coordinates and relativistic equation for the dynamic law of motion in three dimensions. We obtain firstly relativistic transformation for the velocity and acceleration of a particle. Then we substitute them in the relativistic dynamic equation and perform tedious algebraic manipulations. No recourse were made to “general rules for the transformation of 4-tensors”. Formulae obtained were verified in electrodynamics.
1 Introduction

The relativistic mechanics looks in one dimension as [1]

\[ \frac{\dddot{x}}{(1 - \dot{x}^2/c^2)^{3/2}} = F \]  

(1)

where \( \dot{x} = dx/dt \), \( \dddot{x} = d^2x/dt^2 \). Equation (1) is invariant under the Lorentz transformation

\[ x' = \frac{x - vt}{(1 - v^2/c^2)^{1/2}}, \]  

(2)

\[ y' = y, \quad z' = z, \]  

(3)

\[ t' = \frac{t - xv/c^2}{(1 - v^2/c^2)^{1/2}} \]  

(4)

where \( v \) is the parameter that has the meaning of the velocity

\[ v = (v, 0, 0) \]  

(5)

which the inertial frame of reference \( K' \) moves in the inertial frame of reference \( K \). Finding from (2), (4) relativistic transformations of the velocity \( \dot{x} \) and acceleration \( \dddot{x} \) of the body and substituting them in

\[ \frac{\dddot{x}'}{(1 - \dot{x}'^2/c^2)^{3/2}} = F' \]  

(6)

we may verify that the left-hand part of (6) turns exactly into the left-hand part of (1). Hence, we have for the right-hand parts of equations (1) and (6)

\[ F' = F. \]  

(7)

In three dimensions the situation is complicated. The left-hand parts of scalar dynamic equations in \( K' \) are expressed as linear combinations of their left-hand parts in \( K \). This induces respective transformation of the force \( \mathbf{F} \). To find linear relations connecting each of \( F'_x, F'_y \) and \( F'_y \) with \( F_x, F_y \) and \( F_z \) is the aim of the present work. We will proceed in the following sequence.

Firstly, the one-dimensional Lorentz transformation (2)-(4) will be generalized to three dimensions. Then we will find from it the
relativistic transformations of the velocity \( \dot{\mathbf{r}} = d\mathbf{r}/dt \) and acceleration \( \ddot{\mathbf{r}} = d^2\mathbf{r}/dt^2 \), where \( \mathbf{r} = (x, y, z) \). We will substitute them into a three-dimensional generalization of the dynamic equation (1) and after tedious manipulations find the relativistic transformation of \( \mathbf{F} \). Finally, we will apply the result to the system of two electric charges moving with a constant velocity.

2 Three dimensional Lorentz transformation

Let \( \mathbf{v} \) be arbitrarily oriented in space. We have from (2) and (4) for the projection of \( \mathbf{r} \) on the direction of \( \mathbf{v} \)

\[
\mathbf{r}' \cdot \mathbf{v}/v = \gamma(\mathbf{r} \cdot \mathbf{v}/v - vt),
\]

\[
t' = \gamma(t - \mathbf{r} \cdot \mathbf{v}/c^2)
\]

where

\[
\gamma = \frac{1}{(1 - v^2/c^2)^{1/2}}.
\]

By (3) the direction perpendicular to \( \mathbf{v} \) remains unchanged:

\[
\mathbf{r}'_\perp = \mathbf{r}_\perp.
\]

Expanding a vector into the sum of vectors perpendicular and parallel to \( \mathbf{v} \) we get

\[
\mathbf{r} = \mathbf{r}_\perp + \mathbf{r}_\parallel.
\]

This gives, using (8), (11) and (12)

\[
\mathbf{r}' = \mathbf{r}'_\perp + \mathbf{r}'_\parallel = \mathbf{r}'_\perp + (\mathbf{r}' \cdot \mathbf{v}/v)\mathbf{v}/v \\
= \mathbf{r}_\perp + \gamma(\mathbf{r} \cdot \mathbf{v}/v - vt)\mathbf{v}/v \\
= \mathbf{r} + (\gamma - 1)\mathbf{r}_\parallel - \gamma vt.
\]

3 Transformation of velocity

We have from (9) and (13)

\[
dt' = \gamma(dt - \mathbf{r} \cdot \mathbf{v}/c^2),
\]

\[
d\mathbf{r}' = d\mathbf{r} + (\gamma - 1)d\mathbf{r}_\parallel - \gamma v dt.
\]
We find from (14) and (15)
\[
\dot{r}' = \frac{d\dot{r}'}{dt'} = \dot{r} + (\gamma - 1)\dot{r} - \gamma \mathbf{v} = \frac{\dot{r} + \mathbf{v}[(\gamma - 1)\dot{r} \cdot \mathbf{v}/c^2 - \gamma]}{\gamma(1 - \dot{r} \cdot \mathbf{v}/c^2)}. \tag{16}
\]
If \(\mathbf{v}\) is directed along the \(x\)-axis then we may get from (16) and (5)
\[
\begin{align*}
\dot{x}' &= \frac{\dot{x} - v}{1 - \dot{x}v/c^2}, \tag{17} \\
\dot{y}' &= \frac{\dot{y}}{\gamma(1 - \dot{x}v/c^2)}, \quad \dot{z}' = \frac{\dot{z}}{\gamma(1 - \dot{x}v/c^2)}. \tag{18}
\end{align*}
\]
The following useful relation can be obtained from (17) and (18)
\[
\frac{1}{(1 - \dot{r}'^2/c^2)^{1/2}} = \frac{\gamma(1 - \dot{x}v/c^2)}{(1 - r^2/c^2)^{1/2}} \tag{19}
\]
where
\[
\dot{r}^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2. \tag{20}
\]

4 Transformation of acceleration

We have from (14) for the case of (5)
\[
dt' = \gamma(dt - dxv/c^2) = dt\gamma(1 - \dot{x}v/c^2). \tag{21}
\]
Differentiating (17) and using it and (21) we get
\[
\begin{align*}
\ddot{x}' &= \frac{d\ddot{x}'}{dt'} = \frac{d\ddot{x}'}{dt} \frac{dt}{dt'} = \left[\frac{\ddot{x}}{1 - \dot{x}v/c^2} + \frac{(\dot{x} - v)\ddot{x}v/c^2}{(1 - \dot{x}v/c^2)^2}\right] \frac{1}{\gamma(1 - \dot{x}v/c^2)}. \tag{22}
\end{align*}
\]
Using (10) in (22) gives finally
\[
\ddot{x}' = \frac{\ddot{x}}{\gamma(1 - \dot{x}v/c^2)^3}. \tag{23}
\]
Differentiating (18) and using it and (21) we get for a transverse acceleration
\[
\begin{align*}
\ddot{y}' &= \frac{d\ddot{y}'}{dt'} = \frac{d\ddot{y}'}{dt} \frac{dt}{dt'} = \gamma^{-1}\left[\frac{\ddot{y}}{1 - \dot{x}v/c^2} + \frac{\ddot{y}\dot{x}v/c^2}{(1 - \dot{x}v/c^2)^2}\right] \frac{1}{\gamma(1 - \dot{x}v/c^2)}. \tag{24}
\end{align*}
\]
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Relation (24) gives

\[ \ddot{y}' = \frac{1}{\gamma^2 (1 - \dot{x}v/c^2)^2} \left( \ddot{y} + \ddot{x} \frac{\dot{y}v/c^2}{1 - \dot{x}v/c^2} \right). \] (25)

The analogous expression for \( z \) is

\[ \ddot{z}' = \frac{1}{\gamma^2 (1 - \dot{x}v/c^2)^2} \left( \ddot{z} + \ddot{x} \frac{\dot{z}v/c^2}{1 - \dot{x}v/c^2} \right). \] (26)

5 Transformation of \( F_\parallel \)

The three-dimensional relativistic mechanics is [1]

\[ \frac{d}{dt} \left[ \frac{m\dot{r}}{(1 - \dot{r}^2/c^2)^{1/2}} \right] = F. \] (27)

Completing the differentiation in (27) and taking scalar components:

\[
\begin{align*}
\frac{m\ddot{x}}{(1 - \dot{r}^2/c^2)^{1/2}} + \frac{m\dot{x}(\dot{r} \cdot \ddot{r})/c^2}{(1 - \dot{r}^2/c^2)^{3/2}} &= F_x, \\
\frac{m\ddot{y}}{(1 - \dot{r}^2/c^2)^{1/2}} + \frac{m\dot{y}(\dot{r} \cdot \ddot{r})/c^2}{(1 - \dot{r}^2/c^2)^{3/2}} &= F_y, \\
\frac{m\ddot{z}}{(1 - \dot{r}^2/c^2)^{1/2}} + \frac{m\dot{z}(\dot{r} \cdot \ddot{r})/c^2}{(1 - \dot{r}^2/c^2)^{3/2}} &= F_z.
\end{align*}
\] (28)

(29) (30)

Strictly speaking, equation (27) is not Lorentz invariant. However, we may retain the form of (28) in \( K' \) system:

\[
\begin{align*}
\frac{m\ddot{x}'}{(1 - \dot{r}'^2/c^2)^{1/2}} + \frac{m\dot{x}'(\dot{r}' \cdot \ddot{r}')/c^2}{(1 - \dot{r}'^2/c^2)^{3/2}} &= F'_x.
\end{align*}
\] (31)

Substituting (17), (18), (23), (25) and (26) in (31), the left-hand part of (31) can be represented as a linear combination of left-hand parts of equations (28), (29) and (30). This means that retaining the form of (27) we must transform the right-hand part of (27). The component \( F'_x \) of the force is represented as respective linear combination of \( F_x, F_y \) and \( F_z \). Next, we will perform explicitly the procedure mentioned.
Using (19) and (23) in (31) gives
\[
F'_x = \frac{m\ddot{x}}{\gamma^2(1 - \dot{x}v/c^2)^2(1 - \dot{r}^2/c^2)^{1/2}} + 
\frac{m\dot{x}'(\dot{x}'\dddot{x}' + \dot{y}'\dddot{y}' + \dddot{z}')\gamma^3(1 - \dot{x}v/c^2)^3}{c^2(1 - \dot{r}^2/c^2)^{3/2}}. 
\] (32)

Then, substituting (23), (25), (26) and (17)-(18) in the second term of (32):
\[
F'_x = \frac{m\ddot{x}}{\gamma^2(1 - \dot{x}v/c^2)^2(1 - \dot{r}^2/c^2)^{1/2}} + \frac{m(\dot{x} - v)}{c^2(1 - \dot{r}^2/c^2)^{3/2}} \left[ \frac{(\dot{x} - v)\dddot{x}}{(1 - \dot{x}v/c^2)^2} + \frac{\dot{y}}{1 - \dot{x}v/c^2}(\dddot{y} + \dddot{x}v/c^2) + \frac{\dot{z}}{1 - \dot{x}v/c^2}(\dddot{z} + \dddot{x}v/c^2) \right] 
\]
\[
\times \left[ \frac{\dddot{x}}{1 - \dot{x}v/c^2} + \frac{\dot{x}(\dot{x} - v)(1 - \dot{x}v/c^2)}{c^2(1 - \dot{r}^2/c^2)} \right]. 
\] (33)

Firstly, we consider the portion of (33) that contains \(\dddot{x}\). Using in it (10) and (20) gives
\[
\frac{m\dddot{x}}{(1 - \dot{r}^2/c^2)^{1/2}(1 - \dot{x}v/c^2)^2} \left[ 1 - v^2/c^2 + \frac{(\dot{x} - v)^2 + (\dot{y}^2 + \dot{z}^2)(\dot{x} - v)v/c^2}{c^2(1 - \dot{r}^2/c^2)} \right]. 
\] (34)

The expression in quadratic brackets of (34) is
\[
1 - v^2/c^2 + \frac{(\dot{x} - v)^2 - (1 - \dot{r}^2/c^2)(\dot{x} - v)v + (\dot{x} - v)v - \dot{x}^2(\dot{x} - v)v/c^2}{c^2(1 - \dot{r}^2/c^2)} 
\]
\[
= 1 - \dot{x}v/c^2 + \frac{\dot{x}^2 - \dot{x}v - \dot{x}^2(\dot{x} - v)v/c^2}{c^2(1 - \dot{r}^2/c^2)} 
\]
\[
= 1 - \dot{x}v/c^2 + \frac{\dot{x}(\dot{x} - v)(1 - \dot{x}v/c^2)}{c^2(1 - \dot{r}^2/c^2)}. 
\] (35)
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Substituting (35) in (34):

\[
\frac{m\ddot{x}}{(1 - \dot{r}^2/c^2)^{1/2}(1 - \dot{x}v/c^2)} \left[ 1 + \frac{\dot{x}(\dot{x} - v)}{c^2(1 - \dot{r}^2/c^2)} \right] = \frac{m\ddot{x}}{(1 - \dot{r}^2/c^2)^{1/2}} + \frac{m\dddot{x}(\dot{x} - \dot{r}^2v/c^2)}{c^2(1 - \dot{r}^2/c^2)^{3/2}(1 - \dot{x}v/c^2)}.
\]

(36)

We have for members from (33) containing \(\ddot{y}\) and \(\ddot{z}\)

\[
m(\dot{x} - v) \left[ \frac{\dot{x}}{c^2(1 - \dot{r}^2/c^2)^{3/2}(1 - \dot{x}v/c^2)} (\dddot{y} + \dddot{z}) \right].
\]

(37)

Summing (36) and (37) and using \(\dddot{x} + \dddot{y} + \dddot{z} = \dddot{r} \cdot \dddot{r}\) gives

\[
F'_{x} = \frac{m\ddot{x}}{(1 - \dot{r}^2/c^2)^{1/2}} + \frac{m\dddot{x}(\dddot{r} \cdot \dddot{r})}{c^2(1 - \dot{r}^2/c^2)^{3/2}(1 - \dot{x}v/c^2)} - v(\dddot{x} + \dddot{y})
\]

\[
= \frac{m\dddot{x}(\dddot{r} \cdot \dddot{r})}{c^2(1 - \dot{r}^2/c^2)^{3/2}(1 - \dot{x}v/c^2)} - v(\dddot{x} + \dddot{y})
\]

(38)

Using (28) in (38):

\[
F'_{x} = F_x + \frac{m[\dddot{x}(\dddot{r} \cdot \dddot{r}) + v(\dddot{x} + \dddot{y})]}{c^2(1 - \dot{r}^2/c^2)^{3/2}(1 - \dot{x}v/c^2)}
\]

\[
= F_x + \frac{mv[-(\dddot{r} \cdot \dddot{r})(1 - \dot{x}^2/c^2) + \dddot{x}(1 - \dot{r}^2/c^2)]}{c^2(1 - \dot{r}^2/c^2)^{3/2}(1 - \dot{x}v/c^2)}.
\]

(39)

Using (20) in (39):

\[
F'_{x} = F_x - \frac{mv[(\dddot{r} \cdot \dddot{r})(\ddot{y}^2 + \ddot{z}^2)/c^2 + (\ddot{y}\dddot{y} + \dddot{z}^2)(1 - \dot{r}^2/c^2)]}{c^2(1 - \dot{r}^2/c^2)^{3/2}(1 - \dot{x}v/c^2)}
\]

\[
= F_x - \frac{v/c^2}{(1 - \dot{x}v/c^2)} \left\{ \frac{m\ddot{y}}{(1 - \dot{r}^2/c^2)^{1/2}} + \frac{mv(\dddot{r} \cdot \dddot{r})/c^2}{(1 - \dot{r}^2/c^2)^{3/2}} \right\} \dddot{y} +
\]

\[
+ \left( \frac{m\ddot{z}}{(1 - \dot{r}^2/c^2)^{1/2}} + \frac{mv(\dddot{r} \cdot \dddot{r})/c^2}{(1 - \dot{r}^2/c^2)^{3/2}} \right) \dddot{z}
\]

(40)
Using (29) and (30) in (40) we get finally

\[ F'_{x} = F_{x} - (F_{y} \dot{y} + F_{z} \dot{z}) \frac{v^2}{c^2 (1 - \dot{x}v/c^2)}. \]  

(41)

6 Transformation of \( F_{\perp} \)

Using (18), (19), (25) and (17), (23), (26) in

\[ \frac{m \ddot{y}}{(1 - \dot{\mathbf{r}}/c^2)^{1/2}} + \frac{m \dot{y}' (\dot{\mathbf{r}}' \cdot \ddot{\mathbf{r}}')/c^2}{(1 - \dot{\mathbf{r}}/c^2)^{3/2}} = F'_{y} \]  

(42)

we obtain

\[ F'_{y} = \frac{m}{\gamma (1 - \dot{x}v/c^2)} \left\{ \frac{\ddot{y}}{(1 - \dot{\mathbf{r}}/c^2)^{1/2}} + \frac{\dot{y} \dot{x}v/c^2}{(1 - \dot{\mathbf{r}}/c^2)^{1/2}(1 - \dot{x}v/c^2)} \right\} \]

\[ + \frac{\dot{y}}{c^2(1 - \dot{\mathbf{r}}/c^2)^{3/2}} \left[ \frac{(\dot{x} - v)\dot{x}}{1 - \dot{x}v/c^2} + \dot{y} \ddot{y} v/c^2 + \ddot{y} \ddot{x}v/c^2 + \ddot{z} \ddot{z} + \ddot{x} \ddot{z}^2 v/c^2 \right] \]

\[ = \frac{m}{\gamma (1 - \dot{x}v/c^2)} \left\{ \frac{\ddot{y}}{(1 - \dot{\mathbf{r}}/c^2)^{1/2}} + \frac{\dot{y}}{c^2(1 - \dot{\mathbf{r}}/c^2)^{3/2}} \times \right\} \]

\[ \left\{ \frac{(\mathbf{r} - v) \ddot{x}v + \dot{x} \ddot{x}}{1 - \dot{x}v/c^2} + \dot{y} \ddot{y} + \ddot{z} \ddot{z} \right\} \]

\[ = \frac{m}{\gamma (1 - \dot{x}v/c^2)} \left\{ \frac{\ddot{y}}{(1 - \dot{\mathbf{r}}/c^2)^{1/2}} + \frac{\dot{y} (\mathbf{r} \cdot \ddot{\mathbf{r}})/c^2}{(1 - \dot{\mathbf{r}}/c^2)^{3/2}} \right\}. \]  

(43)

Comparing (43) with (29) and using in it (10) we get finally

\[ F'_{y} = F_{y} \frac{(1 - v^2/c^2)^{1/2}}{1 - \dot{x}v/c^2}. \]  

(44)

Similarly for \( z \) component:

\[ F'_{z} = F_{z} \frac{(1 - v^2/c^2)^{1/2}}{1 - \dot{x}v/c^2}. \]  

(45)
7 Relativistic electrodynamics

Let two particles at \((0,0,0)\) and \((x,y,z)\) be at rest in the reference system \(K\). They interact with a force \(\vec{F}\) that can be calculated from some field equations. Next, let these particles move with a constant velocity

\[
\dot{\mathbf{r}} = (\dot{x}, 0, 0).
\]  
(46)

We may calculate the force \(\mathbf{F}\) acted between moving particles from the same field equations. A force can be expanded into the sum of longitudinal and transverse components:

\[
\mathbf{F} = \mathbf{F}_\parallel + \mathbf{F}_\perp.
\]  
(47)

Let us pass to the reference system \(K'\) given by (5) with \(v = \dot{x}\).

\[
v = \dot{x}.
\]  
(48)

Then, according to (41) with (46):

\[
\mathbf{F}'_\parallel = \mathbf{F}_\parallel,
\]  
(49)

according to (45) with (48) and (10):

\[
\mathbf{F}'_\perp = \gamma \mathbf{F}_\perp.
\]  
(50)

By (47), (49) and (50):

\[
\mathbf{F}' = \mathbf{F}'_\parallel + \mathbf{F}'_\perp = \mathbf{F}_\parallel + \gamma \mathbf{F}_\perp.
\]  
(51)

The principle of relativity states that we must have

\[
\mathbf{F}' = \ddot{\mathbf{F}}
\]  
(52)

when \(x' = x\), \(y' = y\) and \(z' = z\). Further we will verify (52) for the case of two electric charges.

We have for two charges \(q_1\) and \(q_2\) at rest

\[
\ddot{\mathbf{F}} = q_1 q_2 \frac{x \mathbf{i}_x + y \mathbf{i}_y + z \mathbf{i}_z}{(x^2 + y^2 + z^2)^{3/2}}.
\]  
(53)
When a charge $q_1$ moves with a constant velocity $\dot{x}$ we must solve equations
\[
\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = -4\pi q_1 \delta(x - \dot{x}t, y, z), \quad (54)
\]
\[
\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 A_x}{\partial t^2} = -\frac{4\pi \dot{x}}{c} q_1 \delta(x - \dot{x}t, y, z). \quad (55)
\]

Using the Lorentz transform (2)-(4) with (48) we may pass in (54) and (55) to reference system $K'$. The left-hand parts of equations (54) and (55) are known to be Lorentz-invariant. In $K'$ the charge is at rest, hence fields $\varphi$ and $A$ do not depend on $t'$. Using the property of $\delta$-function $\delta(|a|x) = \delta(x)/|a|$ we obtain from (54) and (55)
\[
\frac{\partial^2 \varphi}{\partial x'^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = -4\pi q_1 \gamma \delta(x', y, z), \quad (56)
\]
\[
\frac{\partial^2 A_x}{\partial x'^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} = -4\pi q_1 \gamma \frac{\dot{x}}{c} \delta(x', y, z). \quad (57)
\]
Solving equations (56) and (57) we get with (48), (2) and (10)
\[
\varphi = \gamma \frac{q_1}{R}, \quad (58)
\]
\[
A_x = \gamma \frac{v}{c} \frac{q_1}{R}, \quad A_y = 0, \quad A_z = 0, \quad (59)
\]
\[
R = \left[\gamma (x - vt)^2 + y^2 + z^2\right]^{1/2}. \quad (60)
\]
Calculating from (58)-(60) the Lorentz force that acts on a charge $q_2$ which moves in $K$ with the same velocity $v$ we may obtain[2]
\[
\mathbf{F} = q_2 \left[ -\nabla \varphi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} + \frac{1}{c} (\mathbf{v} \times \text{curl} \mathbf{A}) \right]
\]
\[
= q_1 q_2 \frac{\gamma (x - vt) \mathbf{i}_x + \gamma^{-1} (y \mathbf{i}_y + z \mathbf{i}_z)}{\left[\gamma^2 (x - vt)^2 + y^2 + z^2\right]^{3/2}}. \quad (61)
\]
We may isolate in (61) longitudinal $\mathbf{F}_\parallel = F_x \mathbf{i}_x$ and transverse $\mathbf{F}_\perp = F_y \mathbf{i}_y + F_z \mathbf{i}_z$ components according to (47), then substitute them into (51) and use (2) with (10) in the result. This gives
\[
\mathbf{F}' = q_1 q_2 x' \mathbf{i}_x + y \mathbf{i}_y + z \mathbf{i}_z \quad \left[\frac{x'^2 + y^2 + z^2}{(x'^2 + y^2 + z^2)^{3/2}}\right]. \quad (62)
\]
Comparing (62) and (53) for $x = x'$ we confirm[2] formula (52).
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References
