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A PAIR OF INERTIAL FRAMES VERSUS AN INERTIAL FRAME

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Abstract

Without an inertial frame, we are unable to identify and define the force(s) acting upon an object. Without force, we are unable to classify and define an inertial frame(s) to describe the motion of objects. Because the circular definitions of inertia and force, we propose to remove the dynamic aspect of the definition and employ only kinetics in defining what an inertial frame is. Kinetically, we are unable to define an inertial frame, but we can define a pair of inertial frames: a pair of reference frames which have a constant relative velocity with respect to each other. There are infinitely many pairs of inertial frames in the universe, which can be differentiated into separate groups. If a new pair of inertial frames can be formed from an existing pair of inertial frames, then these inertial frames all belong to the same pair-group. The Principle

of Relativity, which states that the laws of physics are the same in all inertial frames, should be only applied to all inertial frames formed from the same pair-group. We utilized the locus of a moving object shown on the geometry graph and applied this pair of inertial frames concept to conclude that relative motion, in some instances, is not entirely relative.

1 Introduction

The definition of an inertial frame has been a problem since Newton first proposed his Laws of Motion. Newton's 1^{st} Law states that an object moving at constant velocity will continue to do so unless acted upon by some net force, but this principle holds true only for reference frames that are inertial frames. However, his 2^{nd} Law defines the force applied to an object as the product of the mass and acceleration of the object as measured from an inertial frame with dynamic property. This argument is circular in that one can only know if a reference frame is inertial by knowing that no force acting on it, but force is also defined relative to an inertial frame. In an attempt to circumvent this, Newton's 1^{st} Law was combined and treated as a special case of Newton's 2^{nd} Law, where $F = 0$, and an inertial frame was defined as a reference frame in which $F = ma$ held true. With the arrival of special relativity, the validity of Newton's 2nd Law was called into question, while Newton's 1^{st} Law still held true, compelling Einstein to redefine an inertial system as one in which an isolated particle, free from interaction, remains at rest or moves uniformly [1]. This is the standard viewpoint of inertial frames accepted by most modern-day physicists. However, it is generally acknowledged that this still leaves a problem in defining an inertial reference frame because it has to be relative to some non-moving body, and even the Earth and the galaxies are in motion [2].

To remedy this complication, we propose to remove the dynamic aspect (force) of the definition, and thus the circular reference, and employ only kinetics in defining what an inertial frame is. We define a pair of inertial frames: a pair of reference frames which have a constant relative velocity with respect to each other.

2 Relativity of Motion

Utilizing this concept in practice is not as straightforward as it seems. In principle, any reference frame associated to an entity that is moving at a constant velocity, i.e. is not accelerating, can be defined as an inertial frame. Suppose there is relative linear acceleration between two reference frames, observers on each of the reference frames would perceive the other to be accelerating and regard their own frame to be a stationary, inertial frame. It would not be possible to determine which one is a truly "inertial frame," because we cannot determine either one to be a fixed reference frame nor can we identify such an absolute frame for comparative verification purposes. The fact that we cannot kinetically detect absolute motion and can only detect relative motion is known as the Relativity of Motion. Thus, kinetically, we are unable to define an inertial frame: a reference frame which moves at a constant velocity with respect to a fixed reference frame. However, we can define a pair of inertial frames: a pair of reference frames which have a constant relative velocity with respect to each other. There are infinitely many pairs of inertial frames in the universe, which can be differentiated into separate groups. If a new pair of inertial frames can be formed from an existing pair of inertial frames, then these inertial frames all belong to the same group. In subsequent references to these, we will utilize the term: pair-group(s). The Principle of Relativity, which states that the laws of physics are the same in all inertial frames, should be only applied to all inertial frames formed from the same pair-group.

If there is relative linear motion or relative circular motion between two reference frames, which are married to two different point particles, then observers on the two reference frames will have the same kinetic description of the other frame. However, if there is relative circular motion between two pairs of inertial frames, then observers on the two different pairs of inertial frames will have different kinetic descriptions of the other pair of frames. Here, we restate the Relativity of Motion with a slight modification: because of no fixed reference frame exists we cannot kinetically detect absolute motion and can only detect relative motion, but in certain cases, the description of the relative motion can be different for observers on different reference frames. We will examine the following four cases in our ensuing discussion: (1) there is relative linear motion between two point particles, (2) there is relative circular motion between two point particles, (3) there is relative linear motion between two pairs of connected point particles and (4) there is relative circular motion between two pairs of connected point particles. We track the locus of a moving object using a geometric program [3].

Case 1: Relative Linear Motion between Two Point Particles

In Fig.1, there is a fixed reference point R, a point particle O that remains stationary, and a point particle O' that moves linearly with respect to reference point R.

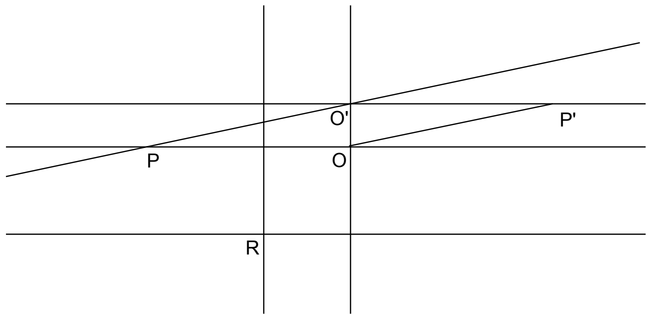

Figure 1: Object O remains stationary while object O' moves linearly with respect to a reference point R. The observer on O will view O' as moving linearly along O'P', and the observer on O' will view O as moving linearly along OP.

When O' moves linearly from O' to P' with respect to R, the observer on O will view O' as having moved from O' to P'. Similarly, the observer on O' will view O as having moved from O to P. Thus, without a fixed reference point, an observer on any point particle cannot determine the nature of their motion, i.e. whether they are stationary or moving along a linear path.

Case 2: Relative Circular Motion between Two Point Particles

In Fig. 2, there is a fixed reference point R, a point particle O that remains stationary, and a point particle O' that moves along a circular path from O' to P' with respect to the reference point R.

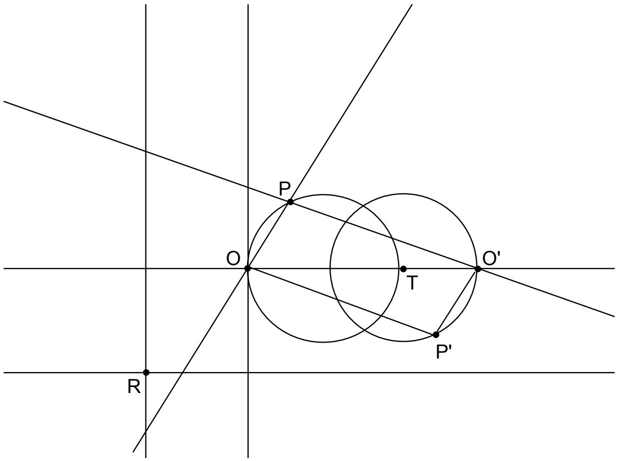

Figure 2: Object O remains stationary while object O' moves in a circle with respect to a reference point R. The observer on O will view O' as moving along a circle, and the observer on O' will view O as moving in a circle.

When the O' moves from O' to P' in a circular trajectory with respect to R, the observer on O will view O' as having moved from O' to P' in a circular trajectory. Similarly, the observer on O' will view O as having moved from O to P in a circular trajectory. Thus, without a fixed reference point, an observer on any point particle cannot determine the nature of their motion, i.e. whether they stationary or moving along a circular path.

Case 3: Relative Linear Motion between Two Pairs of Connected Point Particles

In Fig.3, there is a fixed reference point R, a pair of linked point particles O1 and O2 that remains stationary and a pair of linked point particles O1' and O2' that moves linearly with respect to the reference point R.

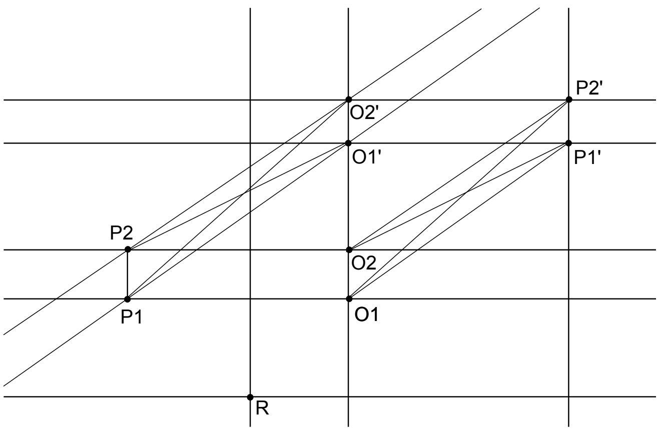

Figure 3: A pair of connected point particles O1 and O2 remains stationary while a pair objects O1' and O2' moves linearly with respect to a reference point R. The observers on O1 and O2 will view O1' and O2' as moving linearly, and the observers on O1' and O2' will view O1 and O2 as moving linearly.

When the O1' and O2' pair moves linearly from O1' and O2' to P1' and P2' with respect to R, the observers on O1 and O2 will view the O1' and O2' pair as having moved linearly from O1' and O2' to P1' and P2'. Similarly, the observers on O1' and O2' will view the O1 and O2 pair as having moved linearly from O1 and O2 to P1 and

P2. Thus, without a fixed reference point, observers on any pair of linked point particles cannot determine the nature of their motion, i.e. whether they are stationary or moving along a linear path.

Case 4: Relative Circular Motion between Two Pairs of Connected Point Particles

In Fig.4, there is a fixed reference point R, a pair of linked point particles O1 and O2 that remains stationary, and a pair of linked point particles O1' and O2' that revolves around the point T with respect to the reference point R.

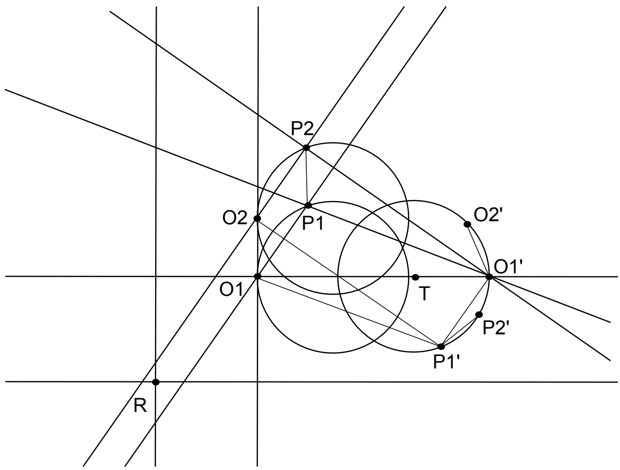

Figure 4: A pair of objects O1 and O2 remains stationary while a pair of objects O1' and O2' revolves around a point T with respect to a reference point R. The observers on O1 and O2 will view O1' and O2' as revolving around a concentric circle, but the observers on O1' and O2' will view O1 and O2 as revolving around circles with different centers.

When the O1' and O2' pair moves from O1' and O2' to P1' and P2' along the circle with the center T with respect to R, the observers on O1 and O2 will view the O1' and O2' pair as having moved from O1' and O2' to P1' and P2' along the same circle. However, the observers on O1' and O2' will view the O1 and O2 pair as having moved from O1 and O2 to P1 and P2, each particle having moved along a different circular path. Here we take note that the observers on the two different pairs of inertial frames will have different kinetic descriptions of the other pair of frames. The frame that undergoes concentric motion is the simpler of the two. Assuming the Law of Parsimony holds true, which is the desirable course in explaining natural phenomena, we can determine whether a system is stationary or moving along the circular path without a fixed reference point, which is a deviation from the notion that all motions are relative without a fixed reference point.

3 Discussion

Here we restate the Relativity of Motion to align with concepts drawn from studying these four cases: we cannot kinetically detect absolute motion and can only detect relative motion, but in certain cases, the description of the relative motion can be different for observers on different reference frames. We observed that if there is relative linear motion or relative circular motion between two reference frames, which are married to two different point particles, then observers on the two reference frames will have the same kinetic description of the other frame. If relative linear motion occurs between two pairs of connected point particles objects, then observers on either pair of objects will also have the same kinetic description of the other pair of objects when assuming their positions to be stationary. Point objects do not exist in this world; they function purely as idealized mathematical situations. Because all objects have a finite size, two points on an object can always be selected to yield a pair of inertial frames, under the assumption that points selected on that object do not undergo heavy deformation with respect to each other. These two points have zero velocity relative to one another, because they are connected in the formation of the object. However, if relative circular motion occurs between two pairs of connected single-point objects, then observers on either pair of objects will have different kinetic descriptions of the other pair of objects when assuming themselves to be stationary.

In physics to understand the theory behind phenomena, it is important to select appropriate reference frame(s) to describe the motion of objects of interest. As long as an object has finite size, which all real objects do, we can select two reference points on the object and build a pair of inertial frames with zero relative velocity. We should bear this in mind when selecting appropriate reference frames.

If there is relative constant velocity between two objects, then a pair of inertial frames attached to each object has same description as two reference points on the other object. It is not possible to tell which object is moving with constant velocity in this experiment, so it is acceptable to choose either object to be stationary based on convenience. For example, if a system only includes two trains with relative constant linear velocity between them, then no one can determine which train is actually moving with respect to the earth, because there is no distinction between the descriptions provided by observers on either train. If the system includes two trains and the earth, then we can build another pair of inertial frames on two posts of the platform and any train that has zero relative velocity with the platform would be treated as being stationary. If two trains pass the platform at the same time with different constant velocities with respect to the platform, then the platform and each train can form a pair of inertial frames. These pairs of inertial frames belong to the same pair-group. Reference frames built on the two trains also form a pair of inertial frames, because these two trains have a constant relative velocity with respect to each other. If two other trains start to depart from the platform with the same acceleration and in the same direction, then these two trains should have zero relative velocity with respect to one another, and reference frames built on these two trains form a pair of inertial frames. This pair of inertial frames does not belong to the same pair-group that the platform belongs to, because there is relative acceleration between these two frames. Affiliation with a certain pair-group bars membership into another pair-group, as they are mutually exclusive.

If there is relative circular motion between two objects, then the pair of inertial frames attached to the object moving in a circle will have a different description than the pair of inertial frames associated with the stationary object. It is possible to tell which object is stationary and which is moving along a circular path, in this experiment through kinetics. For example, if there is constant circular motion between a merry-go-round and a building, then a pair of two inertial frames can be built on two of the merry-go-round horses. A pair of inertial frames can also be built on two posts of the building. With respect to the two building posts, the two horses will be moving along a circular path with a common center. With respect to the two carousel horses, each of the two building posts will be moving along circular paths with different centers. Assuming the more parsimonious of the two descriptions to be real, we should conclude that the building is stationary and that the merry-go-round is moving in a circle.

If a system includes the earth and the sun, then a pair of inertial frames can be attached on the center of the sun and the center of the earth, and another pair of inertial frames can be attached to points on the earth's surface. With respect to the center of the sun and the center of the earth, the two points on the earth's surface will be rotating around a common central axis. With respect to the two points on the earth's surface, the center of the sun and the center of the earth will be moving along circular paths with different centers. Assuming the more parsimonious of the two descriptions to be real, we should conclude that the center of the sun and the center of the earth are stationary and that the two locations on the surface of the earth are moving in a circle.

The same concepts can be extended to a system including a planet, the sun, and a distant star (a pair of inertial frames can be attached to the center of a star and the center of the sun, and a pair of inertial frames can be attached to two selected locations on the planet) and a system including a star, the Milky Way and a distant galaxy (a pair of inertial frames can be attached to the center of the distant galaxy and the center of Milky Way, and a pair of inertial frames can be attached to two points on the star).

4 Conclusion

For simplicity in addressing problems, we can combine the pair of inertial frames and work with a single frame. We can treat objects as point particles to simplify the description of motion. However, a simplification should not make one forget underlying concept of a pair of inertial frames. A reference frame married to any type of motion can essentially be an inertial reference frame, given the appropriate context. Inertia is not an intrinsic, absolute property; it is a product of context and a tool to help us address dynamics.

The motion of any object can be decomposed into linear motion and rotational (circular) motion when described from a reference frame. Kinetically, because no fixed frame exists we are not able to define an inertial reference frame: a reference frame that moves at an absolute constant velocity, but we can define a pair of inertial frames: a pair of reference frames that have a constant velocity with respect to each other. Coupling this pair concept with the Law of Parsimony will help us identify the proper pair(s) of inertial frames to better understand physical phenomena.

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